



# 3rd International Workshop on Linear Systems (LinSys2007)

organized by

**Jochen Trumpf**

**Dates:** Monday 26/02/2007 -- Thursday 01/03/2007

**Venue:** Innovations Theatre, [Innovations Building](#) (#124), [ANU](#), Canberra, Australia

## Program

	Sun 25/02	Mon 26/02	Tue 27/02	Wed 28/02	Thu 01/03
8:00am - - 9:00am		Breakfast*	Breakfast*	Breakfast*	Breakfast*
9:30am - - 10:30am		Jan C. Willems <a href="#">Distributed dissipative systems</a>	Diego Napp <a href="#">An algebraic approach to control theory for PDE</a>	Michael Rotkowitz <a href="#">When is a Linear Controller Optimal?</a>	Paula Rocha <a href="#">Behavioral point-reachability</a>
10:30am -- 11:30am		Matt James <a href="#">An introduction to quantum control</a>	Giorgio Picci <a href="#">Modeling of images by reciprocal processes</a>	Andrea Gombani <a href="#">Minimal symmetric Darlington synthesis: complex and real case</a>	Paolo Rapisarda <a href="#">Recent results in data-driven control</a>
11:30am -- 12:30pm		Uwe Helmke <a href="#">Algorithmic Least Squares Estimation: From Numerical Linear Algebra to Quantum Control</a>	Margreta Kuijper <a href="#">The predictable degree property and a parametrization for annihilators of a behavior over a finite ring</a>	Ian Petersen <a href="#">A complete Kalman decomposition for uncertain linear systems</a>	TBA
12:30pm		Lunch*	Lunch*	Lunch*	Lunch*

-- 2:00pm					
2:30pm - - 3:30pm		Manfred Deistler <a href="#">Modeling High-Dimensional Time Series by Generalized Linear Dynamic Factor Models</a>	Girish Nair <a href="#">Information Flows and Distributed Control</a>	Excursion *	
3:30pm - - 4:30pm		Anders Lindquist <a href="#">Passivity-preserving model reduction by analytic interpolation</a>	Minyue Fu <a href="#">Finite-Level Quantized Feedback Control for Linear Systems</a>	Excursion *	
4:30pm - - 5:30pm		Brian D O Anderson <a href="#">Solution of a distributed linear system stabilisation problem</a>		Excursion *	
5:30pm - - 7:00pm	Reception *			Excursion *	
7:00pm - - 9:00pm	Reception *	Dinner*	Dinner*	Dinner*	

\* for invited speakers only

## **Distributed dissipative systems**

Jan C. Willems

The notion of a dissipative dynamical system generalizes the idea of a Lyapunov function to 'open' dynamical systems. This concept has found applications in diverse areas of systems and control, for example, in stability theory, system norm estimation, and robust control. A central problem that emerges is the construction of a storage function. It is this problem that brought LMI's to the foreground.

The topic of this talk is distributed dissipative systems. First some basic system theoretic concepts, as variable elimination, controllability and observability, are introduced for systems described by linear constant coefficient PDE's, within the behavioral framework.

Subsequently, dissipative systems described by linear PDE's and supply rates that are quadratic expressions in the system variables and their partial derivatives are defined. The dissipation inequality for such systems involves, in addition to the storage function, also the flux. The construction of the storage and the flux reduces to the factorization of polynomial matrices in many variables. This leads straight to Hilbert's 17-th problem regarding the sum-of-squares representation of nonnegative polynomials in many variables. Throughout the talk, Maxwell's equations will be used as the paradigmatic example.

## **An introduction to quantum control**

Matt James

Experimental technique is approaching (or in some applications has reached) the point where it is possible to continuously monitor and manipulate physical systems whose behaviour exhibits quantum mechanical effects. Quantum engineering is emerging as quantum mechanics evolves from its origins in the descriptive science of physics to the prescriptive science of engineering. Quantum control engineering is one aspect of this evolution, and will provide an important methodology for the development of new quantum technologies. In this talk I will describe what quantum control is, some of the underlying fundamentals. I also discuss some of our recent work in this area, which ranges from fundamental concepts to specific applications in quantum optics.

## **Algorithmic Least Squares Estimation: From Numerical Linear Algebra to Quantum Control**

Uwe Helmke

This talk offers an algorithmic approach to constrained least squares estimation. Thus our primary objects of interest are rather the algorithms that allow one to solve least squares estimation tasks and not so much the existence and uniqueness results for the concrete estimation tasks. The nature of such numerical algorithms is strongly influenced by two basic aspects:

1. the geometry and parametrization of the constraint set and
2. the type of dynamics underlying the algorithms.

In this talk we show how such aspects can be used to analyze and tune least squares estimation tasks in quantum control. No knowledge on quantum control is assumed and we review the necessary background.

## **Modeling High-Dimensional Time Series by Generalized Linear Dynamic Factor Models**

Brian D O Anderson, **Manfred Deistler**, Christiane Zinner

Factor models are used to condense high dimensional data consisting of many variables into a much smaller number of factors. The factors represent the comovement between the single time series or underlying nonobserved variables influencing the observations. Here we consider (static and dynamic)

- Principal component models
- Frisch or idiosyncratic noise models
- Generalized linear factor models

## **Passivity-preserving model reduction by analytic interpolation**

Giovanna Fanizza, Johan Karlsson, **Anders Lindquist**, Ryozo Nagamune

Antoulas and Sorensen have recently proposed a passivity-preserving model-reduction method of linear systems based on Krylov projections. The idea is to approximate a positive-real rational transfer function with one of lower degree. The method is based on an observation by Antoulas (in the single-input/single-output case) that if the approximant is preserving a subset of the spectral zeros and takes the same values as the original transfer function in the mirror points of the preserved spectral zeros, then the approximant is also positive real. However, this turns out to be a special solution in the theory of analytic interpolation with degree constraint developed by Byrnes, Georgiou and Lindquist, namely the maximum-entropy (central) solution. By tuning the interpolation points and the spectral zeros, as prescribed by this theory, one is able to obtain considerably better reduced-order models. We also show that, in the multi-input/multi-output case, Sorensen's algorithm actually amounts to tangential Nevanlinna-Pick interpolation.

## **Solution of a distributed linear system stabilisation problem**

Brian D O Anderson

This talk deals with the following problem. Given a real square matrix  $A$ , when is there a real diagonal matrix  $D$  such that  $DA$  has all its eigenvalues with negative real part, and how may such a matrix be constructed? This problem arose in studying how to construct

decentralized feedback control laws used by agents in a two-dimensional formation, where the shape of the formation is to be preserved through a sufficient number of agent pairs each maintaining a prescribed separation, and the open-loop system may be unstable.

A sufficient condition (which in a sense is not "far" from a necessary condition) is obtained, involving the principal minors of  $A$ , and it is fulfilled in the application problem. Some associated open problems are also exposed.

## **An algebraic approach to control theory for PDE**

**Diego Napp, Harry Trentelman**

In this talk, we give a very general overview of the theory of multidimensional systems using a behavioral framework. This framework relies on the idea that control systems are described by equations, but their properties of interest are most naturally expressed in terms of the set of all solutions to the equations. This is formalized by the relatively new notion of the system behaviour (denoted by  $B$ ), due to Willems. We will focus our attention on systems described by partial differential equations with constant coefficients and in the relation between finitely generated modules and these behaviours. This enables the application of the huge and powerful machine of commutative algebra to problems in multidimensional linear systems theory. The main difficulty in using this relation is that it is highly abstract. However we will see how it is possible to give interpretation of some algebraic theory. We will explain the mathematical background needed and show how structural properties such as controllability or autonomy can be describe in terms of some algebraic properties.

In particular I will investigate in some detail the decomposition of a given behavior into the sum of finer components. It is immediately apparent that decomposition is a powerful tool for the analysis of the system properties. It is, indeed, of particular interest, the case of multidimensional systems where a description of the  $nD$  systems trajectories is quite complicated and decomposing the original behavior into smaller components seems to be an effective way for simplifying the systems analysis.

Another motivation for considering this problem is that the problem is also an interesting question from a purely mathematical point of view and allows different approaches.

The autonomous-controllable decomposition has played a significant role in the theory of linear time-invariant systems. Such decomposition expresses the idea that every trajectory of the behavior can be thought of as the sum of two components: a free evolution, only depending on the set of initial conditions, and a force evolution, due to the presence of a soliciting input. In the case of 1D systems, this sum is direct, i.e.

$$B = B_{\text{cont}} + B_{\text{aut}} \text{ and } B_{\text{cont}} \cup B_{\text{aut}} = 0$$

where  $B_{\text{cont}}$  and  $B_{\text{aut}}$  represent the controllable and autonomous part of  $B$  respectively.

However, this decomposition is, in general, not longer direct for  $n \geq 2$ , and we may have that the controllable part of  $B$ , (which is uniquely defined for a given  $B$ ) intersects all possible autonomous parts involved in the controllable-autonomous decomposition [5, 6, 7].

Some of the results I will give are already known, and my main contribution will be a completely new approach to the problem using algebraic geometry, which hopefully opens new ways to tackle open problems in multidimensional systems theory.

Finally we provide an algorithm to effectively solve our problem.

## References

- [1] J. Wood, E. Rogers, and D. H. Owens, Modules and Behaviours in  $nD$  Systems Theory, Mult. Systems and Signal Processing, 2000.
- [2] J. C. Willems, On Interconnections, Control, and Feedback, IEEE Trans. on Auto. Contr.
- [3] U. Oberst, Multidimensional Constant Linear Systems, Acta Applicandae Mathematicae, vol. 20, 1990.
- [4] H. Pillai and S. Shankar, A Behavioural Approach to Control of Distributed Systems, SIAM Journal on Control and Optimization, vol. 37, 1999.
- [5] Wood, Jeffrey and Rogers, Eric and Owens, David H., Controllable and autonomous  $nD$  linear systems, Multidimens. Systems Signal Process., vol. 10, 1999.
- [6] Valcher, Maria Elena, On the decomposition of two-dimensional behaviors, Multidimens. Systems Signal Process., vol. 11, 2000.
- [7] Bisiacco, Mauro and Valcher, Maria Elena, Two-dimensional behavior decompositions with finite-dimensional intersection: a complete characterization, Multidimens. Systems Signal Process., vol. 16, 2005.

## Modeling of images by reciprocal processes

Giorgio Picci

Statistical modeling of images has been the subject of intense research in the past two decades and forms now a vast literature. Most of the literature deals with so-called *Gibbs-Markov* models for random fields borrowed (with some smart twists) from statistical mechanics. Unfortunately these models lead to complicated estimation problems which have to be approached by Monte-Carlo type techniques, such as simulated annealing, MCMC, etc.

We instead propose to use a simple class of stochastic models, known as *reciprocal processes*. These can actually be seen as a special class of G-M random fields and have been well studied in 1-D, especially by Arthur Krener and his collaborators. It can in particular be shown that stationary reciprocal processes admit a *descriptor type* representation of a certain kind which can be seen as a natural non-causal extension of the popular linear state space models used in control and time series analysis. One should

stress that reciprocal processes, in particular stationary reciprocal processes, naturally live in a finite region of the "time" line (or of the plane) and the descriptor models are associated with certain boundary conditions. Estimation and identification of certain classes of these models can in principle be rephrased as a classical problem of *banded extension* for Toeplitz and block-circulant matrices.

## **The predictable degree property and a parametrization for annihilators of a behavior over a finite ring**

**Margreta Kuijper, Raquel Pinto, Jan Willem Polderman**

We consider linear discrete-time systems over the finite ring  $Z_{\{p^r\}}$ . Such systems are relevant in the communications area, notably coded modulation systems. More specifically, syndrome formers and encoders for convolutional codes over  $Z_{\{p^r\}}$  correspond to kernel and image representations of  $Z_{\{p^r\}}$ -behaviors and motivate our work.

In this talk we focus on the predictable degree property for polynomial kernel representations. This property has enjoyed considerable attention in the literature, both system theoretic (Wedderburn, Wolovich, Kailath) and coding theoretic (Forney et al.). In the field case this property is equivalent to row-reducedness and leads to minimality concepts and useful parametrizations of annihilators of a behavior.

How to come up with a sensible definition in the  $Z_{\{p^r\}}$ -case that gives rise to equally useful results? It turns out that this is not straightforward. In the talk we show that the adapted kernel representation that was introduced by Fagnani and Zampieri in the 90's is not suitable for this purpose. Instead we define another, less restrictive representation that fits the bill. We give a construction procedure and derive a parametrization result for annihilators of a  $Z_{\{p^r\}}$ -behavior that parallels the field case.

## **Information Flows and Distributed Control**

**Girish Nair**

This talk concerns the stabilisation of a linear plant with multiple controllers and noisy sensors over a digital network. Well-known results for the centralised case are first recapped. The distributed problem is then formulated, and a nearly necessary and sufficient condition for uniform stabilisability is presented in terms of the feasibility of a number of linear inequalities involving the unstable open-loop eigenvalues of the plant, the various channel data rates and the controllability/observability structures of the plant. This provides a nearly exact characterisation of the region of channel bit rate combinations that permit uniform stability to be achieved. The auxiliary variables introduced in the condition have a natural interpretation as the effective rates of information flow through the network associated with each unstable mode. When channel rates are set to either zero or infinity, this also solves the classical problem of distributed stabilisability over all nonlinear, time-varying, causal policies.

## **Finite-Level Quantized Feedback Control for Linear Systems**

Minyue Fu

In this talk, we study quantized feedback control of discrete-time linear systems using a finite-level quantizer. Motivated by the fact that most feedback communication channels allow a moderate bit rate, we are not particularly concerned with the problem of finding the minimum bit rate of feedback for a given control objective. Instead, we assume that a moderate bit rate is available and our task is to find a practical quantization strategy that achieves a given control objective. We introduce a simple dynamic scaling method and combine it with a known logarithmic quantization method. Using this approach, satisfactory control of linear systems can be achieved using a quantizer with a moderate number of quantization levels. Two main advantages of this approach are

1. It is very easy to implement, and
2. The closed-loop system behaves as if there were no limitation on the number of quantization levels when the state of the system is within a "normal" operating range.

These features are important for practical applications of quantized feedback control.

## **When is a Linear Controller Optimal?**

Michael Rotkowitz

Given a linear system, when is the best controller for it linear as well? This question was a central focus of control theory and game theory several decades ago, and it's having a resurgence. We'll discuss how the answer changes as we consider different cost functions and different information structures. A surprising recent result will be presented, as well as the questions which it leaves open.

## **Minimal symmetric Darlington synthesis: complex and real case**

L. Baratchart, P. Enqvist, **Andrea Gombani**, M. Olivi

Given a square Schur function  $S$ , we consider the problem of constructing a symmetric Darlington synthesis of minimal size. This amounts essentially to finding a stable all-pass square extension of  $S$  of minimal size. The characterization is done in terms of the multiplicities of the zeros. As a special case we obtain conditions for symmetric Darlington synthesis to be possible without increasing the McMillan degree for a symmetric rational contractive matrix which is strictly contractive in the right half-plane. This technique immediately extends to the case where, allowing for a higher dimension of the extension, we require no increase in the McMillan degree. Both the complex and the real case are examined (the last one being that of interest for applications to circuits). Also in this case we obtain sharper results than those existing in the literature.

## **A complete Kalman decomposition for uncertain linear systems**

Ian Petersen

This paper builds on earlier results which studied certain notions of observability and controllability for uncertain systems defined by integral quadratic constraints. These notions were motivated by realization questions for uncertain systems and relate to questions of whether a state is unobservable for all possible uncertainties and whether a state is controllable for some possible uncertainty. Originally, the notion of robust unobservability was characterized in terms of a linear quadratic optimal control problem but later a geometric characterization was obtained. Also, originally, the notion of possible controllability was characterized in terms of a linear quadratic optimal control problem, but in this paper it is also characterized geometrically and this is combined with the geometric characterization of robust unobservability to give a complete Kalman decomposition for uncertain linear systems.

## **Behavioral point-reachability**

Paula Rocha

In this talk we introduce the notion of point-reachability in a behavioral setting. Roughly speaking, this consists in the ability of making every system trajectory reach an arbitrary value at some future instant, independently of its past evolution. Thus, behavioral point-reachability can be viewed as a generalization of the well-known reachability property defined for state space systems. Moreover, this notion is also related to the one of point controllability defined in [1]. This property can be relevant, for instance, in the study of hybrid systems, where the fact that a value (or subspace) in the signal space is attained may trigger a change in the system functioning mode.

[1] J. C. Willems, "Models for dynamics", Dynamics Reported, vol. 2, pp. 171-269, 1989.

## **Recent results in data-driven control**

Ivan Markovsky, **Paolo Rapisarda**

In the classical approach to control, a mathematical model of the plant (in state-space, transfer function, etc. form) and a performance criterion are used in order to come up with a mathematical description of a controller to be used in achieving the control objectives. In the data-driven approach, instead, one trajectory of the plant variables is given, together with the performance criterion; the objective is to compute from this data a suitable control input signal.

In this talk we concentrate on the discrete-time finite-horizon quadratic control problem with prescribed "initial conditions", given in the form of a prefix of a trajectory which needs to be extended over the whole time-interval so as to minimize the cost. We give a solution of this problem, and we illustrate some results of our data-driven investigation.

Among these is an intrinsic justification of the optimality of the state-feedback control input law which is prominent in the state-space approach; we show that this fact can be established from first principles, and is not only a consequence of the use of state-space representations.