# A note on duality of control and observation

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## I. Extended abstract

Control and observation are in general not dual problems. While the *pole placement parts* of the classical static state feedback control problem on the one hand and the classical full state observer problem on the other hand are dual in the classical sense (in the context of linear time-invariant plants in input/state/output form), this is not true for the full state feedback control/state observer problem, where the interconnection structure is included in the consideration.

More precisely, the above classical duality consists of a duality pairing between systems where the system

$$\dot{x} = Ax + Bu,$$
$$y = Cx$$

is paired with the dual system

$$\dot{x} = A^*x + C^*y = A^dx + B^du^d,$$
$$u = B^*x = y^d = C^dx,$$

plus the usual duality of the linear maps $(A - BF)$ and $(A - BF)^* = (A^* - F^*B^*) = (A^d - JC^d)$. In fact, it is well known that the above duality pairing between systems can also be interpreted as a duality of triplets of linear maps between suitable trajectory spaces combined with time reversal, see e.g. [1], [3]. This duality between feedback and output injection gives rise to a rich geometrical theory of invariant subspaces that has been very widely studied including in the author's PhD thesis [3].

In this talk I will show that a variant of classical duality holds in the context of reference tracking control where the reference is generated by an exosystem and the general observer problem. In both cases there results an internal model principle, discovered by Bruce A. Francis and W. Murray Wonham for the control problem [5] and by the author, Harry L. Trentelman and Jan C. Willems for the observer problem [4]. These two principles are dual in a precise category theoretic sense.

It is hence somewhat surprising that such a duality result does *not* seem to hold for the general state feedback control/state observer problem. This can be seen if one tries to augment the above duality to encompass all the signals in the controller resp. observer interconnection. It becomes immediately obvious that the numbers of arrows do not match, ruling out a duality within the category of LTI systems.

It appears that a general, categorical notion of duality between control and observation can only be formulated

1) within one and the same feedback loop (*internal* duality), or,
2) in terms of swapping the roles of the plant and the controller/observer (*external* duality).

In both cases a somewhat artificial extension with additional signals is required to make this work.

An interesting consequence is that a solution to the control problem for a *given* plant does not necessarily yield a "dual" solution to the observer problem for the *same* plant. This appears to be a known fact in the literature on time-varying systems [2].

## References

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