Frequency, temperature and salinity variation of the permittivity of Seawater

Ram Somaraju and Jochen Trumpf

Abstract

With the emergence of unmanned marine robots, underwater communication systems have received much attention in recent years. To successfully develop radio wave based communication solutions, it is essential to understand properties of electromagnetic wave transmission in seawater. These properties are determined by the frequency variation of the permittivity of seawater. Existing models for the permittivity of saline water are empirical ones that best fit experimental data. We propose a physically realistic model, similar to the one used in plasma physics, for the variation of the dielectric constant of water with varying frequencies and salinities. Our model is in excellent agreement with existing empirical fits for frequencies between 1 and 256 GHz. We use this model to study the propagation of electromagnetic waves in seawater. We explain that large propagation distances would be possible at MHz frequencies if the conductivity of seawater decreases at small field strengths due to the hydrogen bonding of water molecules. However, we were unable to experimentally verify any reduction in the conductivity of seawater.

Index Terms

Permittivity, Underwater radio propagation, Electromagnetic propagation in plasma media, Attenuation

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Ram Somaraju and Jochen Trumpf are with the Department of Information Engineering, Research School of Information Sciences and Engineering, Building 115, The Australian National University, Canberra, ACT 0200, Australia and National ICT Australia Limited, Locked Bag 8001, Canberra, ACT 2601, Australia.

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I. INTRODUCTION

EXISTING systems for underwater communication largely depend on acoustic technologies. However, acoustic communication is riddled with problems including time-varying multipath propagation and large latencies. Therefore, Al-Shammaa *et. al.* claim that radio communication is a viable alternative [1]. To understand the properties of radio wave propagation in seawater it is essential to know the frequency variation of seawater's relative permittivity because the rate of attenuation of plane electromagnetic waves is a function only of the relative permittivity of the medium.

However, existing models used for the permittivity of seawater are empirical ones that best fit experimental data and are not based on a sound physical model. We propose a model for the permittivity of seawater that is similar to the one used for ionic plasmas. In the following, we briefly review the general theory of polarization of dielectrics including Debye's theory of molecular relaxation. This is followed by a description of models that are currently being used to determine the permittivity of fresh and sea water. We then explain our model, and continue with an evaluation of the model including implications on electromagnetic wave propagation. As will be shown in section VI the predictions of this model disagree with the results of Al-Shammaa *et. al.* [1] but agree with the measurements we calculated.

II. POLARIZATION

Any dielectric substance placed in an electric field undergoes polarization, which involves the appearance of bound charges on the surface of the dielectric. Polarization is defined as the dipole moment per unit volume and it may be divided into two categories: *induced and orientation polarization* [2], [3].

Debye investigates the contribution of these two forms of polarizations to the relative permittivity of a dielectric substance [4]. In section IV we will introduce *atomic polarization* leading to a theory more appropriate for seawater.

A. Debye's Theory

In the book Polar Molecules [4], Debye explains the relationship between relative permittivity and the frequency of electromagnetic waves in a dielectric medium. Debye assumes that the molecules are free and do not interact with each other [2], [3] and that the polarization of the dielectric consists of induced and orientation components.

When a static electric field $\mathbf{E}$ is applied to a dielectric, the induced component of polarization is assumed to have no inertia and almost instantaneously attains a value of $\mathbf{P}_i = \epsilon_0(\epsilon_\infty - 1)\mathbf{E}$. However, the orientation polarization rises exponentially to reach a maximum value of $\mathbf{P}_o = \epsilon_0(\epsilon_s - 1)\mathbf{E} - \epsilon_0(\epsilon_\infty - 1)\mathbf{E}$ at $t = \infty$. Therefore, for a static electric field $\mathbf{E}$, the total polarization reaches a maximum value of $\mathbf{P} = \epsilon_0(\epsilon_s - 1)\mathbf{E}$. The time-constant τ of the exponentially increasing orientation polarization is called the *relaxation time*. Also, ϵ_s and ϵ_∞ are the static and infinite frequency relative permittivities of the dielectric and ϵ_0 is the permittivity of free space. It depends on the temperature of the substance and is independent of the nature of the electric field and the time of application of the field. Note that other parameters such as viscosity and pressure influence ϵ_r. But, this variation is not studied in the paper. Based on these assumptions, it can easily be shown (see [2], [3]) that the frequency dependance of the relative permittivity ϵ_r may be written as

$$\epsilon_r(\omega) = \epsilon_\infty + \frac{\epsilon_s - \epsilon_\infty}{1 + j\omega\tau} \tag{1}$$

Here, ω is the angular frequency of oscillation of the electric field. It should be noted that the terms in the above equation are functions of the temperature T of the substance and therefore it is more appropriate to write equation (1) as

$$\epsilon_r(\omega, T) = \epsilon_\infty(T) + \frac{\epsilon_s(T) - \epsilon_\infty(T)}{1 + j\omega\tau(T)} \tag{2}$$

1) Dielectric properties of real molecules: Debye's model is based on the assumption that there are no intermolecular interactions and this simple model does not accurately predict the permittivity of real dielectrics. Several dielectrics may be better modelled using the Cole-Cole model [5] which states that the relative permittivity is given by

$$\epsilon_r(\omega, T) = \epsilon_\infty(T) + \frac{\epsilon_s(T) - \epsilon_\infty(T)}{1 + (j\omega\tau(T))^{1-h}} \tag{3}$$

where $0 \leq h \leq 1$. This is an empirical model and does not really have a physical basis. There is however an interpretation of the Cole-Cole model as the result of a distribution of relaxation times rather than a single relaxation time. Several fits have been proposed for the permittivity of both sea and fresh water based on both Debye and Cole-Cole models as explained in the following section.

III. EXISTING MODELS FOR SEA AND FRESH WATER PERMITTIVITY

An extensive set of experimental measurements [6]–[9] are available for the permittivity of fresh water. The dielectric properties of fresh water may be modelled using equ. 1 for frequencies up to 100 GHz [10]. However, for higher frequencies a double-Debye model is found to be more appropriate. The double-Debye model is based on the assumption that there exists a second polarization process with a different relaxation time and is given by the equation

$$\begin{aligned}\epsilon_r(\omega, T) = \epsilon_\infty(T) + \frac{\epsilon_s(T) - \epsilon_1(T)}{1 + j\omega\tau_1(T)} \\ + \frac{\epsilon_1(T) - \epsilon_\infty(T)}{1 + j\omega\tau_2(T)}\end{aligned} \tag{4}$$

The widely used equations of Liebe *et. al*. [10] are based on such a double-Debye model. Liebe *et. al.* [10] claim that their model may be used for frequencies up to 1THz and may be extended up to 30THz by the inclusion of two Lorentzian terms. Double-Debye fits for fresh water are found in several other papers including Stogryn *et. al.* [11] and Meissner *et. al.* [12].

Until recently, comprehensive models based on extensive experimental measurements were not freely available for seawater. Descriptions used for seawater until the early 1990s consisted of works of Stogryn [13] and Klein [14]. However, in the last decade and a half several single and double Debye type models were developed by Ellison *et. al.* [15], Stogryn *et. al.* [11] and Meissner *et. al.* [12]. The double-Debye model used by Meissner *et. al.* [12] and Stogryn *et. al.* [11] is similar to the fresh water model with the addition of the effect of conductivity on the dielectric constant and may be written as

$$\begin{aligned}\epsilon_r(\omega, T, S) = \epsilon_\infty(T, S) + \frac{\epsilon_s(T, S) - \epsilon_1(T, S)}{1 + j\omega\tau_1(T, S)} \\ + \frac{\epsilon_1(T, S) - \epsilon_\infty(T, S)}{1 + j\omega\tau_2(T, S)} + j\frac{\sigma(T, S)}{\epsilon_o\omega}\end{aligned} \tag{5}$$

Here, S is the salinity of seawater in parts per thousand (ppt). Ellison *et. al.* [15] use a single Debye model to fit to experimental data. All these models evaluate the functional dependance of the terms in equation (5) on the salinity and temperature by fitting polynomial, rational or exponential functions to experimental data. For example, Meissner *et. al.* [12] use the fit

$$\epsilon_s = \epsilon_s(T, 0) \cdot \exp(b_0 S + b_1 S^2 + b_2 T S) \tag{6}$$

for the static relative permittivity of seawater. Here, $\epsilon_s(T, 0)$ is the static relative permittivity of fresh water and the constants b_i are evaluated by fitting the best curve to experimental data. By the author's own admission, there is no physical basis for the model (equation (6)) used. In addition to using the dielectric model of fresh water Ellison *et. al.* [15], Stogryn *et. al.* [11] and Meissner *et. al.* [12] respectively use 30, 13 and 12 parameters that are determined from experimental data to predict the variation of all the terms in equation (5) with temperature and salinity. In contrast, our model is not only physically realistic but also uses only two additional parameters to describe the dielectric behavior of seawater.

IV. MODEL OF SEAWATER PERMITTIVITY

Seawater has several dissolved salts and is therefore a good conductor. However, increased conduction is not the only phenomenon that occurs when salts are dissolved in water. The ions are hydrated to varying extents (see [16]–[18]).

The *hydration number* is defined as the number of water molecules in the immediate vicinity of the ion. It is based on the dynamical behavior of the water molecules in solution that move with the ion as a unit [17]. This should be distinguished from the *coordination number* of the ion which is the number of molecules in the immediate neighborhood of the ion. The coordination number depends on the distance of the water molecules from the ion [17]. It is bigger than the hydration number and includes all the molecules that are hydrogen bonded to the molecules in the immediate vicinity of the ion.

The model we propose here assumes that the Debye model of Stogryn *et. al.* [11] is adequate for fresh water. However, we develop a physically realistic model for the variation in the permittivity of seawater with varying salinities and temperatures. There are three basic differences between sea and fresh water that need to be considered in order to develop this model.

- The conductivity of water increases with the addition of ions and the increase in conductivity is approximately proportional to the number of ions.
- The extent of polarization due to the displacement of bound charges (i.e. induced and orientation polarization) in seawater depends on its salinity due

  to the presence of ions. Therefore, ϵ_s, ϵ_∞ and τ are functions of seawater's salinity.

  The static relative permittivity ϵ_s, of seawater reduces because all the water molecules that are in the vicinity of an ion orient themselves with respect to the ion. We assume that these molecules do not contribute to the orientation polarization of seawater. We further assume that the number of water molecules that orient themselves about the dissolved ions is directly proportional to the number of ions. Hence, we would expect ϵ_s to decrease linearly with increasing salinity. This assumption is in accordance with the model of Ellison *et. al.* [15] and furthermore seems reasonable based on the physical intuition given above.

  The effect of the ions on the induced polarizability is difficult to analyze. Firstly, each ion will have a different absorption spectrum in the infrared region and will contribute different amounts to induced polarization. Further, the ions will affect the magnitude of induced polarization of water molecules. However, if the concentration of ions is small, these effects may be ignored.

  Also, the time constant τ should not be affected by the addition of ions. This is because τ is based on the inertial properties of orientation polarization and we are assuming that the water molecules that are oriented about the ions do not contribute to the orientation polarization. Also, the inertial forces on the water molecules that are not near the ions should not be effected significantly by the presence of ions.

- In addition to induced and orientation polarization, there exists a third kind of polarization in seawater. Non-uniform distribution of free ions in the water will result in *atomic polarization*, $\mathbf{P}_f$. The contribution of $\mathbf{P}_f$ to polarization has to be taken into account in calculating the relative permittivity.

A. Polarization of seawater

The model we propose here is based on the one used for gaseous plasmas which is composed of positive and negative ions, electrons and also neutral atoms [19]. The total polarization of seawater, $\mathbf{P}$ may be written as $\mathbf{P} = \mathbf{P}_b + \mathbf{P}_f$. Here, $\mathbf{P}_b$ is the polarization due to the displacement of bound charges in water molecules (i.e. induced and orientation polarization) and $\mathbf{P}_f$ is due to the displacement of ions inside water (i.e. atomic polarization). We can write $\mathbf{P}_b = \epsilon_0 \chi \mathbf{E}$, where $\chi = \epsilon_b - 1$ and

$$\begin{aligned} \epsilon_b(\omega, T, S) = \epsilon_\infty(T) &+ \frac{\epsilon_1(T) - \epsilon_\infty(T)}{1 + j\omega\tau_2(T)} \\ &+ \frac{\epsilon_s(T)(1 - \alpha(T)S) - \epsilon_1(T)}{1 + j\omega\tau_1(T)} \end{aligned} \tag{7}$$

This is similar to equation (4) that is used for fresh water with the small but significant additional term $-\alpha(T)S$ in accordance with the assumption that the static relative permittivity of seawater decreases linearly with increasing salinity. The remaining terms in this equation are assumed to be the same as the one used by Stogryn *et. al.* [11] to model fresh water.

1) *Evaluation of* $\mathbf{P}_f$*:*

We make the following assumptions in deriving a model for the variation of atomic polarization with frequency.

- Seawater is composed of water and several dissolved ion types, indexed by i, with mass m_i and charge q_i. m_i is the total mass of the i^{th} type of ion and all the water molecules in the hydration shell of this type of ion.
- The drift velocity of the water molecules is zero and the drift velocities of all other ions are measured with respect to water.
- The density of ions is small and so collisions between ions may be ignored and only collisions between neutral water molecules and ions are significant.

If collisions are ignored, the rate of change of the drift velocity of the i^{th} type of ion $\mathbf{v}_i$ may be written as

$$\begin{aligned} N_i m_i \left(\frac{\partial \mathbf{v}_i}{\partial t} + \mathbf{v}_i \cdot \nabla \mathbf{v}_i \right) &= N_i q_i \mathbf{E} + N_i q_i \mathbf{v} \times \mathbf{B} \\ &\quad + N_i m_i \mathbf{g} - \nabla p_i \end{aligned} \tag{8}$$

where N_i is the number of ions of type i per unit volume, $p_i = N_i k T_i$ is the pressure and $N_i m_i \mathbf{g}$ gives the gravitational force. Though not shown explicitly for ease of notation, both N_i and m_i are functions of temperature and salinity. For wavelengths that are large compared to atomic dimensions, the pressure gradient and the nonlinear $\mathbf{v}_i \cdot \nabla \mathbf{v}_i$ terms may be ignored [19]. Furthermore, gravitational force is small compared to the force due to the electric field and can therefore be ignored. Collisions are incorporated into equation (8) by adding a damping term that is proportional to $\mathbf{v}_i$ and an effective collision rate ω_i^{eff} and we get,

$$\begin{aligned} N_i m_i \left(\frac{\partial \mathbf{v}_i}{\partial t} \right) &= N_i q_i \mathbf{E} + N_i q_i \mathbf{v}_i \times \mathbf{B} \\ &\quad - N_i m_i \omega_i^{eff} \mathbf{v}_i \end{aligned} \tag{9}$$

If we define the drift displacement $\mathbf{r}_i$ of the i^{th} ion by

$$\mathbf{v}_i = \frac{\partial \mathbf{r}_i}{\partial t} \tag{10}$$

then we can write

$$\mathbf{P}_f = \sum_i N_i q_i \mathbf{r}_i \tag{11}$$

Also, equation (8) may be re-written in terms of drift displacement as

$$\begin{aligned} N_i m_i \frac{\partial^2 \mathbf{r}_i}{\partial t^2} &= N_i q_i \mathbf{E} + N_i q_i \frac{\partial \mathbf{r}_i}{\partial t} \times \mathbf{B} \\ &- N_i m_i \omega_i^{eff} \frac{\partial \mathbf{r}_i}{\partial t} \end{aligned} \tag{12}$$

For waves with exponential dependance of the form $\exp\{j(\mathbf{kr} - \omega t)\}$, this equation may be written as

$$(j\omega)^2 \mu_i \mathbf{r}_i = q_i(\mathbf{E} + j\omega \mathbf{r}_i \times \mathbf{B}) \tag{13}$$

where $\mu_i = m_i\{1 + j(\omega_i^{eff}/\omega)\}$. Substituting equation (13) into (11), and ignoring the contribution of the magnetic field, which tends to be small compared to that of the electric field in non-magnetic materials, we get

$$\mathbf{P}_f = -\sum_i \frac{N_i q_i^2}{\mu_i \omega^2} \mathbf{E} \tag{14}$$

B. The relative permittivity

Now, the total displacement field $\mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P}_f + \mathbf{P}_b$. Using equations (7) and (14) we get

$$\begin{aligned} \epsilon_r(\omega, T, S) = \epsilon_\infty(T) &+ \frac{\epsilon_s(T)(1 - \alpha(T)S) - \epsilon_1(T)}{1 + j\omega\tau_1(T)} \\ &+ \frac{\epsilon_1(T) - \epsilon_\infty(T)}{1 + j\omega\tau_2(T)} - \sum_i \frac{N_i q_i^2}{\epsilon_0 \mu_i \omega^2} \end{aligned} \tag{15}$$

Substituting the value of μ_i from equation (13) we get

$$\begin{aligned} \epsilon_r(\omega, T, S) = \epsilon_\infty(T) &+ \frac{\epsilon_s(T)(1 - \alpha(T)S) - \epsilon_1(T)}{1 + j\omega\tau_1(T)} \\ &+ \frac{\epsilon_1(T) - \epsilon_\infty(T)}{1 + j\omega\tau_2(T)} \\ &- \sum_i \frac{c_i}{\epsilon_0 \omega^2 (1 + j\omega_i^{eff}/\omega)} \end{aligned} \tag{16}$$

where $c_i = \frac{N_i q_i^2}{m_i}$.

It is however difficult to calculate and measure ω_i^{eff} for the individual ions. Therefore, we assume that the effective collision rate is the same for all the ions and is equal to ω^{eff}. We can then rewrite equation (16) as

$$\begin{aligned} \epsilon_r(\omega, T, S) = \epsilon_\infty(T) &+ \frac{\epsilon_s(T)(1 - \alpha(T)S) - \epsilon_1(T)}{1 + j\omega\tau_1(T)} \\ &+ \frac{\epsilon_1(T) - \epsilon_\infty(T)}{1 + j\omega\tau_2(T)} \\ &- \frac{c(T,S)}{\epsilon_0 \omega^2 (1 + j\omega^{eff}(T,S)/\omega)} \end{aligned} \tag{17}$$

where $c(T,S) = \sum_i c_i$. Both $c(T,S)$ and $\omega(T,S)$ are functions of both Salinity and temperature because the number N_i of ions of type i in solution and the mass m_i of i^{th} type of ion along with its hydration shell are functions of temperature and salinity.

1) Evaluation of $\alpha(T)$, $c(T,S)$ and $\omega^{eff}(T,S)$: $\alpha(T) \cdot S$ is equal to the fraction of water molecules that are oriented towards ions in solution. Let the concentration of the i^{th} type ion in water be ρ_i parts per thousand. Now, some fraction, say β_i, of these ions will be dissociated and these are the only ions that contribute towards the reduction of the static permittivity of water. The number of such ions N_i in 1 Kg of solution is given by

$$N_i = \frac{\beta_i \rho_i}{\nu_i} \text{ moles} \tag{18}$$

Here, ν_i is the atomic mass of the i^{th} type ion. If the coordination number of this ion is k_i, then the total number of water molecules that are oriented about the i^{th} type of ion is $N_i k_i$. Therefore the fraction of water molecules, $\alpha(T) \cdot S$ that are oriented about all the ions in solution is given by

$$\alpha(T) \cdot S = \sum_i \frac{N_i k_i}{1/0.018} = \sum_i \frac{0.018 \beta_i \rho_i k_i}{\nu_i} \tag{19}$$

Note that we used the fact that the molecular mass of water is 18 g/mole in deriving the last equation.

To evaluate $c(T,S)$, we need to know the hydration number and not the coordination number of an ion. Let the hydration number of the i^{th} type of ion be h_i and $A_v = 6.023e23$ be Avogadro's number. Then the mass of the i^{th} type of ion and all the water molecules in its hydration shell is $m_i = (\nu_i + 0.018h_i)/A_v$. Using this and equation (18) we get

$$\begin{aligned} c(S,T) &= \sum_i c_i \\ &= \sum_i \frac{N_i q_i^2}{m_i} \\ &= \sum_i \frac{A_v^2 \beta_i \rho_i q_i^2}{\nu_i(\nu_i + 0.018h_i)} \end{aligned} \tag{20}$$

Finally, because the ionic conductivity of water $\sigma(T,S) \approx c(T,S)/\omega^{eff}(T,S)$ we can calculate the ratio of $\omega^{eff}(T,S)$ to $c(T,S)$ using the well established values of ionic conductivity of water.

However, the hydration number, coordination number and percentage of dissociation are difficult to ascertain accurately from experimentation. For instance, the experimental values for the hydration number of Na+ ions varies from 4-8 [16]. Therefore it was decided that $\alpha(T)$ and $c(T,S)$ should be determined using experimentally

Ion Type	Density (ppt)
Cl^-	19.135
Na^+	10.76
SO_4^{-2}	2.712
Mg^{2+}	1.294

TABLE I

DENSITY AND MOLALITIES OF COMPONENTS OF SEAWATER WITH SALINITY S = 35 PPT

measured values of the permittivity and then the resulting value for the hydration and coordination numbers be compared to existing predictions.

V. RESULTS

It is difficult to experimentally measure the permittivity of seawater for a varying range of frequencies because it is extremely lossy. Therefore, it was decided that the validity of the model be ascertained by generating pseudo-data from the empirical fits to experimental data mentioned in section III. Figure 1 compares the real and imaginary parts of permittivity of seawater of our model with the fits of Stogryn *et. al.* [11], Meissner *et. al.* [12], Ellison *et. al.* [15] and Wentz *et. al.* [20] for various values of salinity and temperature.

It is clear from these figures that our model is in excellent agreement with these fits for frequencies between 1 and 256 GHz. The maximum deviation of our model from the fits of Stogryn *et. al.* [11] and Meissner *et. al.* [12], is 7.6% and 6.9% respectively in the frequency range 1-256GHz. The fits of Stogryn *et. al.* [11] and Meissner *et. al.* [12] differ by as much as 11.5% in the same frequency range. Further verification of the model comes from the fact that the values of $\alpha(T)$ and $c(T,S)$ obtained by fitting best curves to pseudo-data are the right order of magnitude. We get,

$$\alpha(T) = 0.00314 \text{ ppt}^{-1} \tag{21}$$

$$c(T,S) \approx 1\times 10^{12}\cdot S \text{ C}^2/\text{Kg} \tag{22}$$

It should be noted that no temperature dependance was detected for both $c(T,S)$ and $\alpha(T)$. To check that these values are as expected, we need to know the composition of seawater. Table I shows the densities of four components of seawater that have the highest concentrations [21].

We note that the only significant elements are Sodium and Chlorine. Because we are only interested in the order of magnitude of the hydration numbers we assume that seawater only consists of these two components and from the calculated value of α we get

$$\beta(k_{Cl}+k_{Na}) = 11.3 \tag{23}$$

Here, β is the degree of dissociation of NaCl and k_{Cl} and k_{Na} are the coordination numbers of Cl and Na respectively. These numbers are as expected [17]. Similarly it can be shown that $c(T,S)$ is of the right order of magnitude.

However, more experimental results over a wider range of frequencies are required to completely validate the model and get an accurate value for $c(T,S)$. Such experiments are particularly important for frequencies greater than a few hundred gigahertz. High frequency measurements are needed to accurately compute $c(T,S)$ because ω^{eff} is of the order of 1THz and we require measurements in this frequency range to accurately determine $c(T,S)$.

Also, at smaller frequencies seawater behaves as a conductor and this can easily be seen by taking the low frequency limit of equation (17). Then the permittivity reduces to

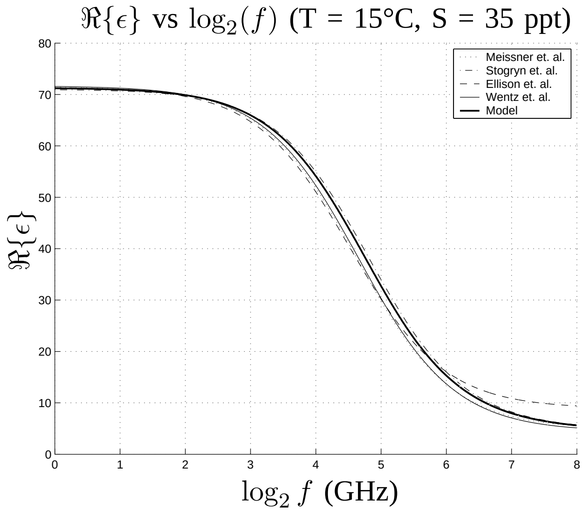
$$\epsilon_r(T,S) = \frac{c(T,S)}{j\epsilon_0\omega\omega^{eff}(T)} = \frac{\sigma(T,S)}{j\epsilon_0\omega} \tag{24}$$

This is what one would expect for a good conductor. In fact the low frequency limit of the empirical equation (5) is identical to the one used in this model. It is therefore safe to assume that the model is valid for low frequencies.

VI. IMPLICATIONS FOR THE PROPAGATION OF ELECTROMAGNETIC WAVES

Our model for the permittivity of seawater assumes that it is independent of the applied electric field strength and is only a function of the temperature and salinity of seawater and the frequency of the electromagnetic wave. Therefore the rate of attenuation of an electromagnetic wave in seawater, which depends only on its permittivity, is not a function of the distance from a transmitting antenna. This is in accordance with the classical literature: the articles [22]–[26] indicate that no such change in attenuation occurs as the distance from the transmitting antenna increases even if the antennas are insulated [26], [27]. Therefore, we expect the range in seawater to be comparable to the skin depth, which is of the order of 0.3 m at a frequency 1 MHz if we assume the conductivity of seawater to be 4 s/m [28].

However, Al-Shammaa *et. al.* claim that radio communication over a distance of 100m is possible at MHz frequencies in seawater [1]. Al-Shammaa *et. al.* [1] further claim that as the distance from the transmitting antenna increases, the rate of attenuation of electromagnetic waves reduces greatly. In fact, figures 13 and 14 in [1] indicate that there is minimal attenuation once the distance from the transmitting antenna increases beyond


$\Re\{\epsilon\}$ vs $\log_2(f)$ (T = 15°C, S = 35 ppt)
$\Re\{\epsilon\}$
$\log_2 f$ (GHz)
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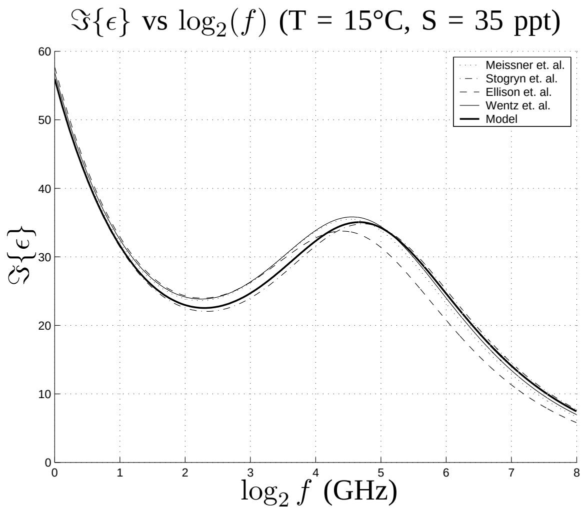
Meissner et. al.
Stogryn et. al.
Ellison et. al.
Wentz et. al.
Model

(a)


$\Im\{\epsilon\}$ vs $\log_2(f)$ (T = 15°C, S = 35 ppt)
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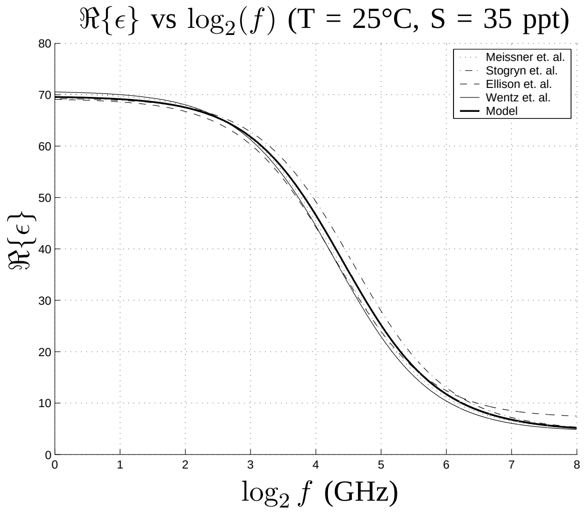
Meissner et. al.
Stogryn et. al.
Ellison et. al.
Wentz et. al.
Model

(b)


$\Re\{\epsilon\}$ vs $\log_2(f)$ (T = 25°C, S = 35 ppt)
$\Re\{\epsilon\}$
$\log_2 f$ (GHz)
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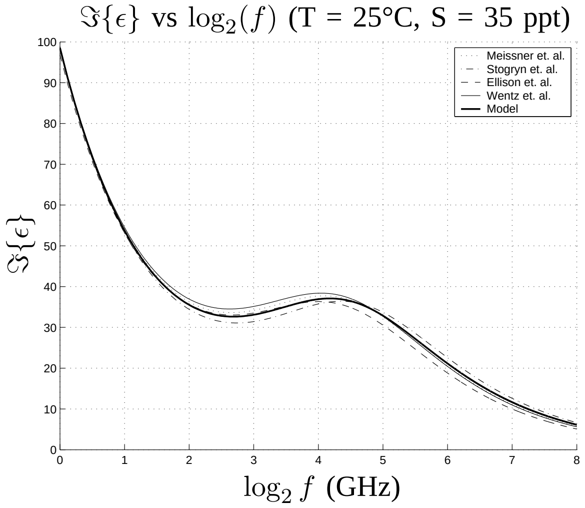
Meissner et. al.
Stogryn et. al.
Ellison et. al.
Wentz et. al.
Model

(c)


$\Im\{\epsilon\}$ vs $\log_2(f)$ (T = 25°C, S = 35 ppt)
$\Im\{\epsilon\}$
$\log_2 f$ (GHz)
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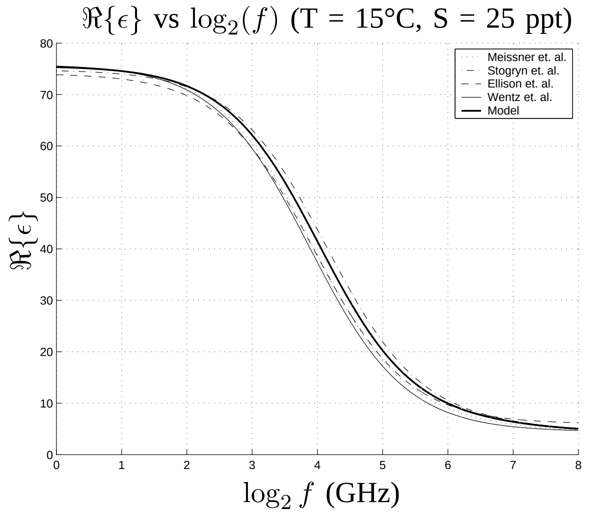
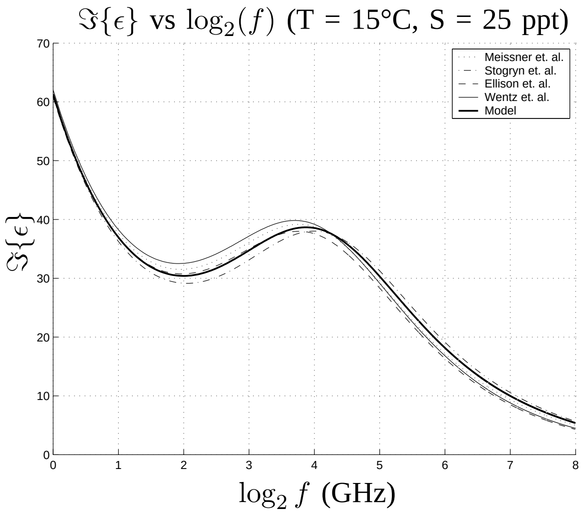
Meissner et. al.
Stogryn et. al.
Ellison et. al.
Wentz et. al.
Model

(d)


$\Re\{\epsilon\}$ vs $\log_2(f)$ (T = 15°C, S = 25 ppt)
$\Re\{\epsilon\}$
$\log_2 f$ (GHz)
0
10
20
30
40
50
60
70
80
0
1
2
3
4
5
6
7
8
Meissner et. al.
Stogryn et. al.
Ellison et. al.
Wentz et. al.
Model

(e)


$\Im\{\epsilon\}$ vs $\log_2(f)$ (T = 15°C, S = 25 ppt)
$\Im\{\epsilon\}$
$\log_2 f$ (GHz)
0
10
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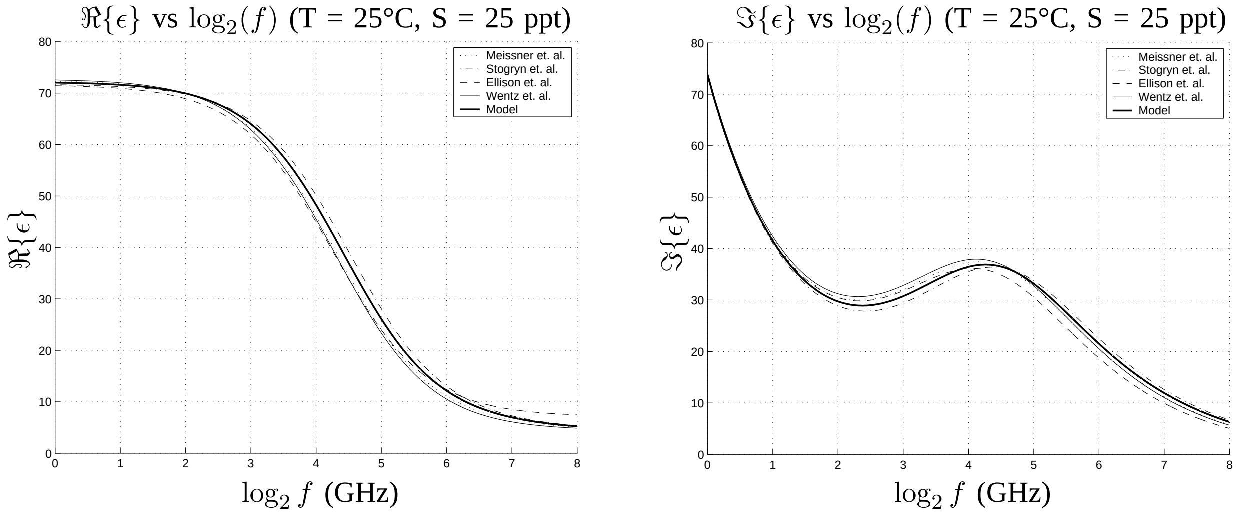
Meissner et. al.
Stogryn et. al.
Ellison et. al.
Wentz et. al.
Model

(f)



(g) (h)

Fig. 1. Real and Imaginary parts of permittivity as a function of frequency

3-4 meters. Al-Shammaa *et. al.* [1] explain this reduction in attenuation with increased distance by claiming that the conduction current losses may be ignored once the distance from the transmitting antenna becomes large.

We believe that such a change in attenuation of electromagnetic waves in seawater could only occur if seawater behaves differently at small electric field strengths and hence at large propagation distances from a transmitting antenna. If the conductivity of seawater decreases at small electric field strengths, then, as the distance from the transmitting antenna increases, the amplitude of the transmitted electromagnetic wave would reduce and therefore we would see a reduced rate of attenuation. One possible explanation as to why seawater might be a poor conductor at small field strengths is as follows.

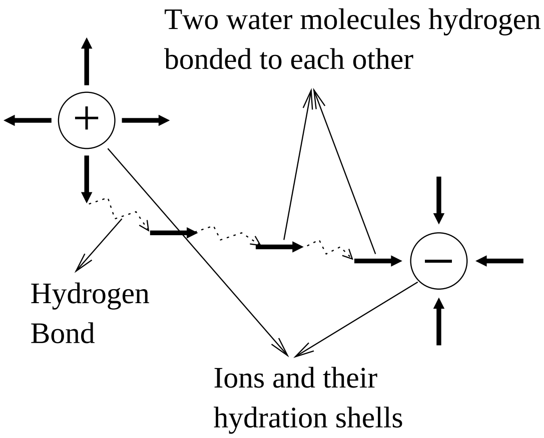


Fig. 2. Ions *bonded* to each other through hydrogen bonded water molecules

A positive and a negative ion may be *bonded* to each other through water molecules that are hydrogen bonded to each other (See figure 2). These bonds, if they do exist, will be extremely weak and easy to break apart. Therefore at high electric field strengths, with forces acting in opposite directions on positive and negative ions, these bonds might be broken apart and we would get free positive and negative ions. However, for small electric field strengths, there would be no free ions to conduct and therefore the conductivity might decrease drastically. This would particularly be the case at higher frequencies because with alternating fields, the time available to break these weak bonds would be shorter. It is well known that the conductivity of seawater is not constant above a certain frequency. Gabillard *et. al.* [25] show that if the conductivity was constant for high frequencies then yellow light would only be visible up to 29 cm underwater. But we can see a yellow lamp much further than 29 cm in seawater.

It is hence conceivable that the rate of attenuation decreases with decreasing field strength. It could well be that such a reduction has not been detected previously because one could not measure extremely small electric field strengths until recently. However, with better measuring equipment available now one might be able to detect small field strengths.

We decided to experimentally verify if the conductivity of seawater changes by using the setup shown in figure 3. We measured the amplitude of $V1$ and $V2$ using a lock-in amplifier to calculate the impedance of salt water from the ratio $V1$ and $V2$. We used a

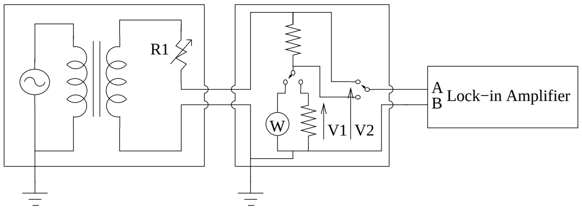


Fig. 3. Experimental setup to measure the conductivity of saltwater

Princeton Applied Research EG & G 5210 amplifier at a frequency of 50 KHz and a Stanford Research Systems SR844 amplifier at a frequency of 1 MHz. It was decided that it was unnecessary to use a Wheatstone bridge circuit because it is not essential that the impedance be measured accurately. We were only interested in measuring large changes in impedance as only this would explain the large differences in the rates of attenuation. As a control experiment, at 1 MHz the water cell was also replaced by a 820 Ω carbon resistor.

The electric field strength applied to the water cell was reduced by increasing the resistance of the variable resistor $R1$. All the components were shielded inside grounded metal boxes to reduce the effects of external noise. We detected no change in the conductivity of salt water at 50 KHz. The smallest voltage applied to the water cell was $600nV$ at 50 KHz. The cell is 5 cm long and if we assume a uniform electric field then the smallest field applied was $12\mu V/m$.

However, at 1 MHz we initially detected a change in the ratio of the voltages $V1$ and $V2$. Exactly the same change in ratio was also present in the control experiment, where we replaced the water cell with a resistor. We concluded that this effect was due to capacitative and inductive coupling between poorly shielded wires. After shielding the wires properly we did not detect a change in the ratios of the two voltages and hence no change in the conductivity of seawater down to a voltage of 30 μV at 1 MHz. If we assume a uniform electric field then the smallest field strength applied was 1.5 mV/m.

We can conclude from this experiment that the conductivity of seawater does not change for electric field strengths as small as $12\mu V/m$ at a frequency of 50 KHz or $1.5mV/m$ at a frequency of 1 MHz and hence the rate of attenuation does not change for these field strengths. We currently have no plausible explanation for the large propagation range observed by Al-Shammaa *et. al.* [1].

VII. CONCLUSION

In this paper we derived a physically realistic model for the frequency variation of the relative permittivity of seawater for varying salinities and temperatures. The model derived is in excellent agreement with existing empirical fits to experimental data. Also, the model uses only two parameters that need to be determined from experimental data as opposed to more than 10 parameters used by most empirical fits. Furthermore, the remaining parameters in our model have a physical interpretation and could hence theoretically be determined by independent experiments. Moreover, because our model has a physical foundation, we are confident that it is valid over a wider parameter (frequency, temperature and salinity) range and can be used for extrapolation in regions where no experimental data is available.

This model however does not predict large propagation distances for electromagnetic waves in seawater in the frequency range of a few Megahertz as measured by Al-Shammaa et. al. [1]. We believe that the only possible explanation for these large propagation distances is that the conductivity of seawater changes at small field strengths due to hydrogen bonding in water. However, we measured no change in conductivity for electric fields as small as $12\mu V/m$ and $1.5mV/m$ at frequencies of 50 KHz and 1 MHz respectively.

REFERENCES

[1] A. I. Al-Shammaa, A. Shaw, and S. Saman, “Propagation of electromagnetic waves at mhz frequencies through seawater,” *IEEE Trans. Antennas Propagat.*, vol. 52, no. 11, pp. 2843–2849, Nov. 2004.

[2] A. Chelkowski, *Dielectric Physics*. Elsiver Scientific Publishing Company, 1980.

[3] E. H. Nora, W. E. Vaughan, A. Price, and M. Davies, *Dielectric properties and molecular behaviour*, T. Sugden, Ed. Van Nostrand Reinhold Company Ltd., 1969.

[4] P. Debye, *Polar molecules*. Dover, 1929.

[5] K. S. Cole and R. H. Cole, “Dispersion and absorption in dielectrics,” *Journal of chemical physics*, vol. 9, pp. 341–351, April 1941.

[6] J. Hasted, S. Hussain, A. Frescura, and J. Birch, “The temperature variation of the near millimetre wavelength optical constants of water,” *Infrared Physics*, vol. 27, no. 1, pp. 11–15, 1987.

[7] D. Lide, Ed., *Handbook of Chemistry and Physics*, 74th ed. CRC Press, 1993, ch. 6, p. 10.

[8] D. Archer and P. Wang, “The dielectric constant of water and debye-huckel limiting law slopes,” *Journal of Physical Chemical Referrence Data*, vol. 19, p. 371, 1990.

[9] R. L. Kay, G. Vidulich, and K. S. Pribadi, “A reinvestigation of the dielectric constant of water and its temperature coefficient,” *Journal of Chemical Physics*, vol. 73, no. 2, pp. 445–447, February 1969.

[10] H. J. Liebe, G. A. Hufford, and T. Manabe, “A model for the complex permittivity of water at frequencies below 1thz,” *International Journal of Infrared and Millimeter Waves*, vol. 12, no. 7, pp. 659–675, 1991.

[11] A. Stogryn, H. Bull, K. Rubayi, and S. Iravanvhy, “The microwave dielectric properties of sea and fresh water,” GenCorp Aerojet, Azusa, Ca. 91702, Tech. Rep., 1995.

[12] T. Meissner and F. Wentz, “The complex dielectric constant of pure and sea water from microwave satellite observations,” *IEEE transactions on geosciense and remote sensing*, vol. 43, no. 29, Sept 2004.

[13] A. Stogryn, “Equations for calculating the dielectric constant of saline water,” *IEEE transactions on microwave theory and Techniques*, vol. 19, pp. 733–736, August 1971.

[14] L. A. Klein and C. T. Swift, “An improved model for the dielectric constant of sea water at microwave frequencies,” *IEEE transactions on antennas and propagation*, vol. 25, no. 1, pp. 104–111, January 1977.

[15] W. Ellison, A. Balana, G. Delbos, K. Lamkaouchi, L. Eymard, C. Guillou, and C. Prigent, “New permittivity measurements of seawater,” *Radio Science*, vol. 33, no. 3, pp. 639–648, May-June 1998.

[16] D. T. Richens, *The chemistry of aqua ions*. John Wiley & Sons, 1997.

[17] E. Clementi, *Determination of liquid water structure, coordination number for ions and solvation for biological molecules*. Springer-Verlag, 1976.

[18] J. Burgess, *Ions in solution*. Horwood Publishing, 1999.

[19] H. G. Booker, *Cold plasma waves*. Martinus Nijhoff Publishers, 1984.

[20] F. Wentz and T. Meissner, “Amsr ocean algorithm (version 2),” Remote Sensing Systems (http://www.remss.com), Santa Rosa, CA, Tech. Rep., 1999.

[21] K. Turekian, *Oceans*. Englewood Cliffs, NJ: Prentice-Hall Inc, 1976.

[22] R. Dunbar, “The performance of a magnetic loop transmitter receiver system submerged in the sea,” *The Radio and Electronic Engineer*, vol. 42, no. 10, pp. 457–463, October 1972.

[23] I. Bogie, “Conduction and magnetic signalling in the sea - a background review,” *The Radio and Electronic Engineer*, vol. 42, no. 10, pp. 447–452, October 1972.

[24] R. King and G. Smith, *Antennas in Matter*. The M.I.T. Press, 1981, ch. 9.

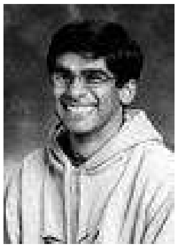
[25] R. Gabillard, P. Degauque, and J. Wait, “Subsurface electromagnetic telecommunication- a review,” *IEEE Trans. Commun. Technol.*, vol. COM-19, no. 6, pp. 1217–1228, 1971.

[26] J. Wait, “Insulated loop antenna immersed in a high conductivity medium,” *J. Res. Nat. Bur. Stand.*, vol. 59, no. 2, pp. 133–137, August 1957.

[27] R. King, “Theory of the terminated insulated antenna in a conducting medium,” *IEEE Trans. Antennas Propagat.*, vol. 12, no. 3, pp. 305 – 318, May 1964.

[28] M. Tucker, “Conduction signalling in the sea,” *The Radio and Electronic Engineer*, vol. 42, no. 10, pp. 453–456, October 1972.

Jochen Trumpf (M’04) received the Dipl.-Math. and Dr. rer. nat. degrees in mathematics from the University of Würzburg, Germany, in 1997 and 2002, respectively. He is working as a Research Fellow for the Department of Information Engineering in the Research School of Information Sciences and Engineering at The Australian National University, Canberra, Australia, and is currently seconded to National ICT Australia Ltd. His research interests include observer theory and design, linear systems theory and optimisation problems in digital communication.



Ram Somaraju received B.Sc. and B.E. degrees in Physics and Computer Systems engineering from the University of Auckland, New Zealand, in 2001 and 2003, respectively. He is currently pursuing a Ph.D. degree in the Department of Information Engineering in the Research School of Information Sciences and Engineering at The Australian National University, Canberra, Australia. His research interests include underwater radio communications, MIMO communication and uncertainty principles for communication channels.