Improved Robustness of a Robot Pose Filter Using the Direct Product Geometry on SE(3)

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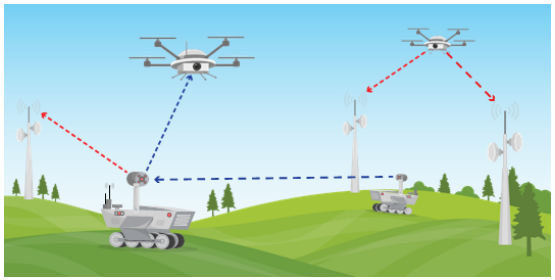
Outline

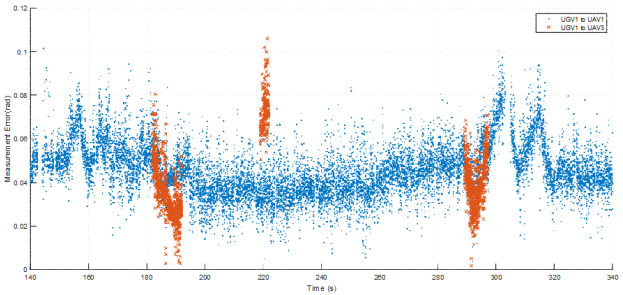
1. Problem description
2. The filtering framework
3. The surprising observation
4. Conclusion

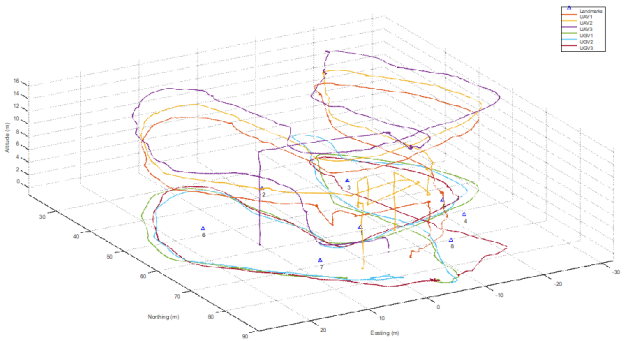
Wider context: Collaborative localisation







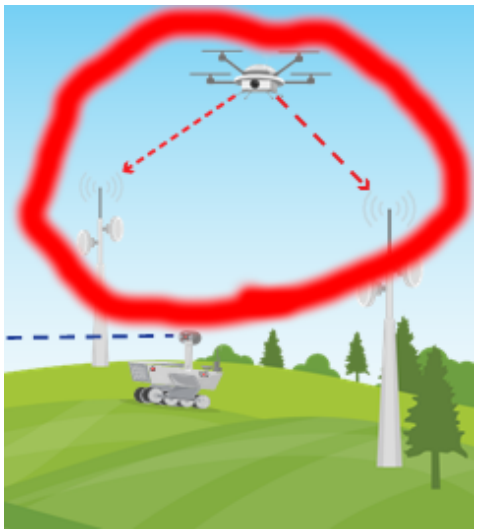




Jack Henderson, Collaborative Localisation of Autonomous Vehicles, PhD thesis, ANU 2024.

Estimation of robot pose using landmark bearings

* stationary landmarks
* we have a map of the landmarks (their global positions)
* measure bearings to the landmarks (e.g. with a camera)
* problem is highly nonlinear
* measurement frequency is very low (in the order of 0.5 - 1 Hz)



Typical GPS-INS filters have orientation errors in the order of 5 degrees (without landmark bearings).

Minimum energy filtering - propagation step

$\dot{x} = f(x, u - B\delta)$ (dynamic constraint)

$J_t(x, \delta) = \frac{1}{2}\|x(0) - \hat{x}_0\|^2_{P_0^{-1}} + \frac{1}{2}\int_0^t \|\delta(\tau)\|^2 \mathrm{d}\tau$

$V(x, t) = \min_{\delta|_{[0,t]}} J_t(x, \delta),\ V(x(0), 0) = \frac{1}{2}\|x(0) - \hat{x}_0\|^2_{P_0^{-1}}$

$\hat{x}(t) = argmin_x V(x, t)(t)$

An alternative to stochastic filters, originally proposed by Mortensen in 1968. For LTI systems and the usual noise assumptions it yields the same filter formulas as the Kalman filter (see e.g. Sontag's book).

Continuous discrete filter - update step for relative measurements

$$y = h(x; l) + D\epsilon$$

$$V^+(x,t) = V(x,t) + \tfrac{1}{2}\|y - h(x;l)\|^2_{W^{-1}}$$

$$\hat{x}^+(t) = \operatorname{argmin}_x V^+(x,t)(t)$$

If the state space is a Lie group and the measurement lies in a homogeneous space, we can compute a second order approximation to this filter, the so-called GAME filter (see Zamani/Trumpf CDC2019 for the continuous-discrete version). For kinematics on the Lie group SO(3) it yields the same formulas as the MEKF (plus some curvature correction terms that Markley knew about but neglected).

GAME filter for robot pose - propagation step

$$\dot{X} = X \begin{bmatrix} \Omega \\ V \end{bmatrix}^{\wedge}, \; X(0) = X_0$$

$$U = \begin{bmatrix} \Omega \\ V \end{bmatrix}^{\wedge} + (B\delta)^{\wedge}, \; B = \begin{bmatrix} B_\omega & 0_{3\times 3} \\ 0_{3\times 3} & B_v \end{bmatrix}, \; \delta = \begin{bmatrix} \delta_\omega \\ \delta_v \end{bmatrix}$$

$$U_\omega = \Omega + B_\omega \delta_\omega, \; U_v = V + B_v \delta_v$$

$$\dot{\hat{X}} = \hat{X} U, \quad \hat{X}(0) = \hat{X}_0$$

$$\dot{P} = -\bar{U}^\top P - P\bar{U} + BB^\top$$

$$\bar{U} \triangleq \begin{bmatrix} (U_\omega)^\times & 0 \\ (U_v)^\times & (U_\omega)^\times \end{bmatrix}, \quad \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}^\times \triangleq \begin{pmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{pmatrix}$$

GAME filter for robot pose - update step

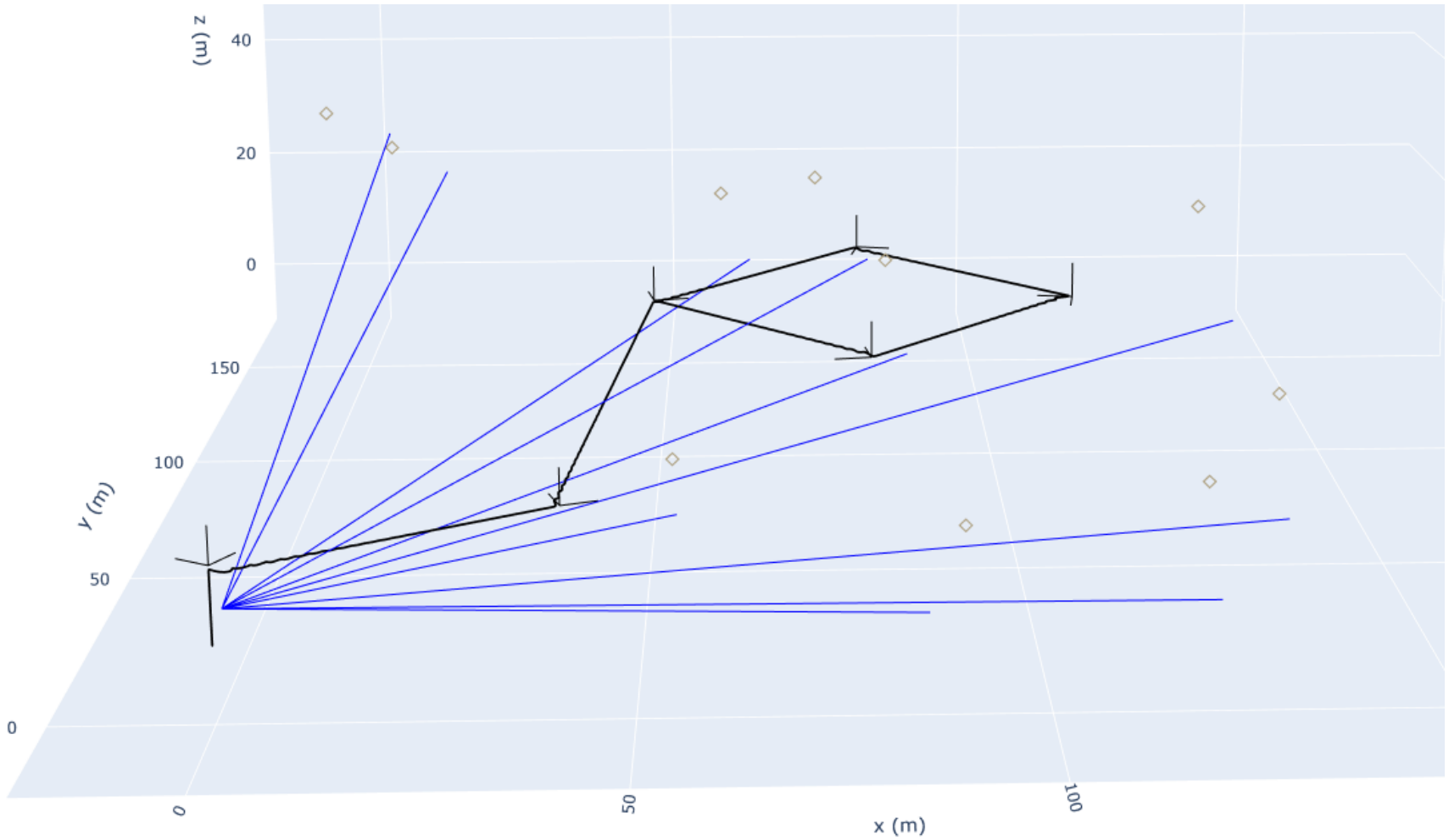
$$J(X) = \frac{1}{2}\|DX^{-1}\bar{p}_i\|^2$$

$$D \triangleq \begin{bmatrix} \frac{z_\theta^\perp (z_\theta^\perp)^\top}{\sigma_\theta} + \frac{z_\psi^\perp (z_\psi^\perp)^\top}{\sigma_\psi} & 0_{3\times 1} \\ 0_{1\times 3} & 0 \end{bmatrix}, \quad \begin{aligned} \theta_i^m &= \theta_i + \delta_\theta \\ \psi_i^m &= \psi_i + \delta_\psi \end{aligned}$$

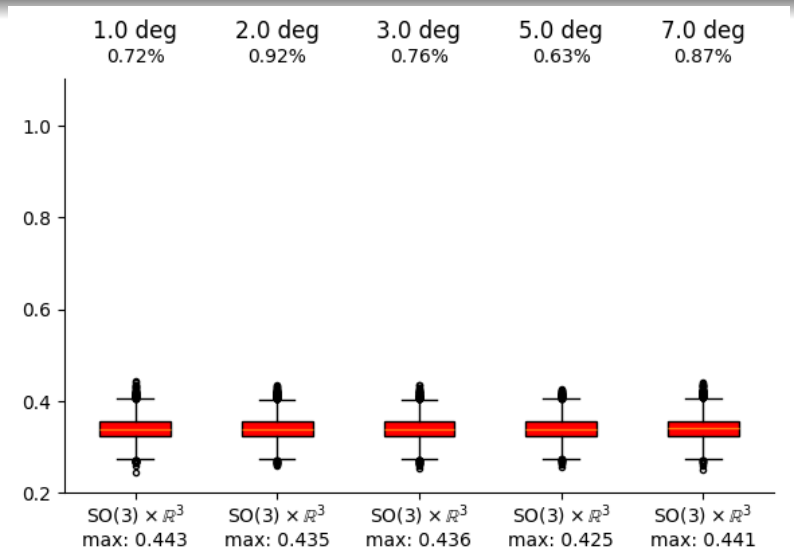
$$z_\theta^\perp \triangleq \begin{bmatrix} -\sin(\theta_i^m) \\ \cos(\theta_i^m) \\ 0 \end{bmatrix}, \; z_\psi^\perp \triangleq \begin{bmatrix} -\cos(\theta_i^m)\sin(\psi_i^m) \\ -\sin(\theta_i^m)\sin(\psi_i^m) \\ \cos(\psi_i^m) \end{bmatrix}$$

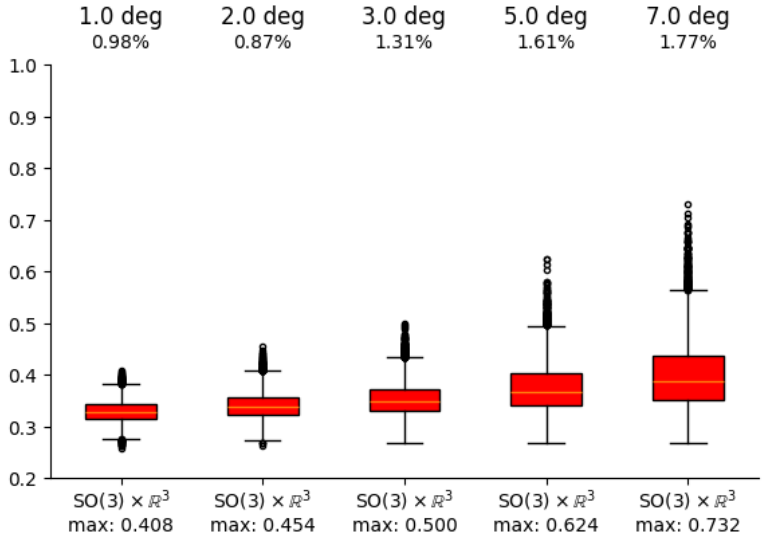
$$P^+ = (P^{-1} + H^{-1})^{-1},$$
$$\hat{X}^+ = \hat{X}\exp_{\hat{X}}(-P^+ g),$$

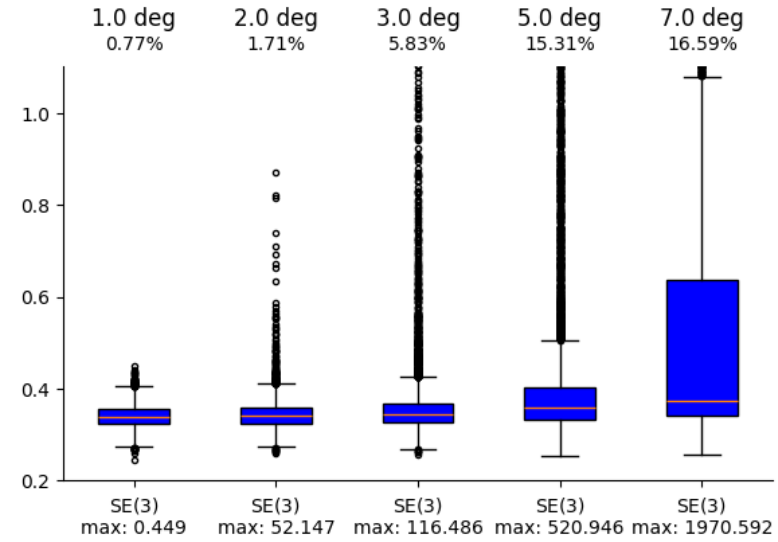
The simulated scenario

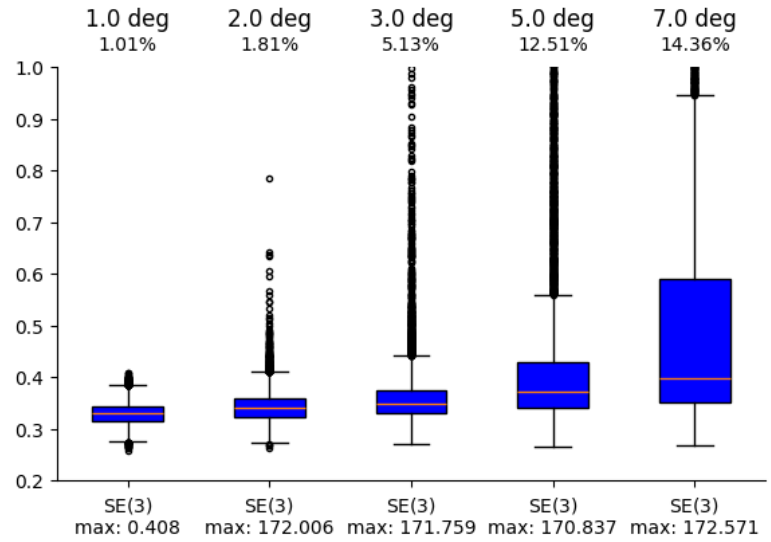


Monte Carlo simulation results

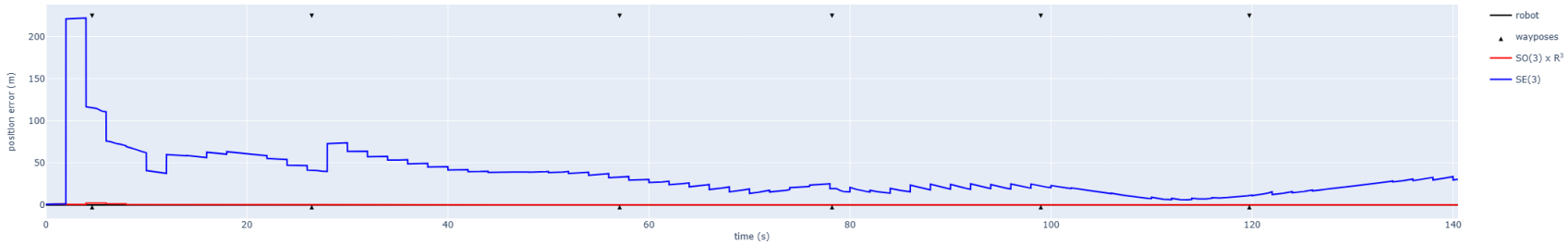


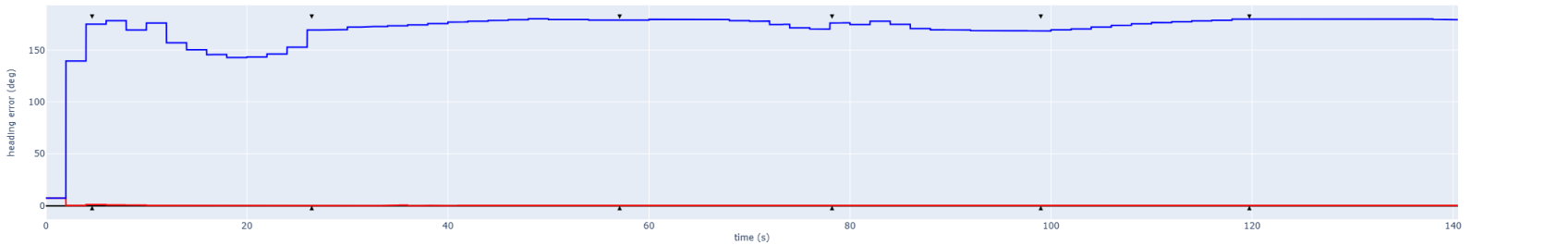


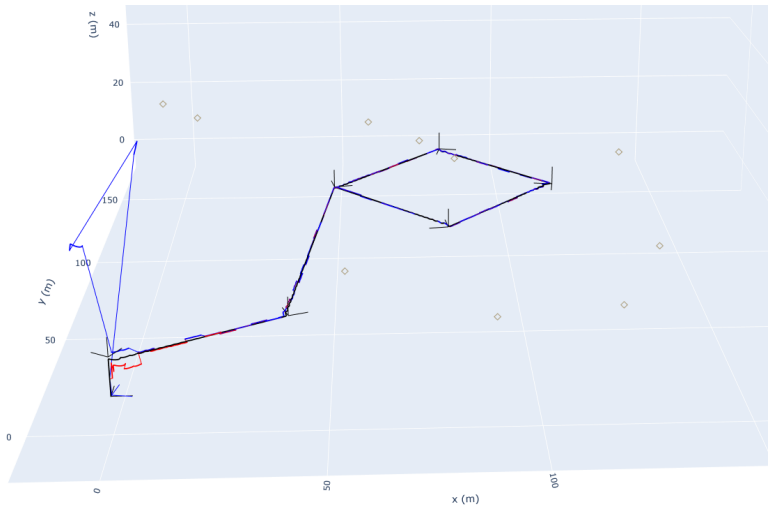




The typical failure mode







Using an invariant metric for SE(3) leads to large overshoot reminiscent of the helical trajectories observed in the hand-eye coordination problem

Conclusion

- For a Lie group state space and a sufficiently nonlinear problem, the choice of Riemannian metric in a GAME filter implementation matters for filter robustness
- This is somewhat surprising since at the same time, the filters have a large convergence radius
- We need to study the global approximation properties of second-order Taylor expansions of cost functions on Lie groups
- Of particular interest is the case of reductive Lie groups with a compact factor
- There might be a connection to Morse theory because the failure mode appears to be related to convergence to an unstable critical point

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Thank you.