Exploiting symmetry in observer design for flying robots

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ANU

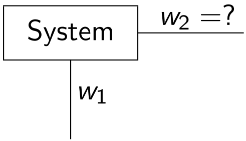
July 2018

Outline

1. Observer theory
2. Kinematic systems with symmetry
3. Motivating examples from robotics and computer vision
4. Observer design for kinematic systems with symmetry

The observation problem

Given a set of *variables* (signals) whose interaction is described by a known *dynamical system* and given *measurements* of some of the variables, can you provide *good estimates* of (other) variables in the system? How?



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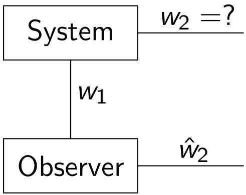


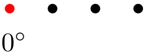
Can you do it with an *observer*?

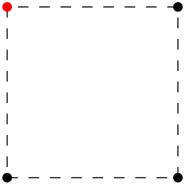
Observer = system interconnected with the observed system
Estimate = value of variable in the observer

Ingredients for a theory of observers

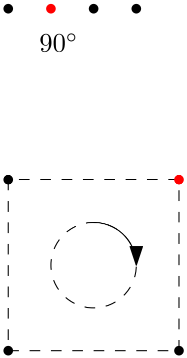
- Model class for the observed system (incl. measurement model)
- What makes an estimate a *good* estimate?
- Is the problem solvable (*observability*)?
- Model class for candidate observers
- Is the problem still solvable (*existence*)?
- How do you recognize a solution (*characterization*)?
- How do you build an observer (**construction/design**)?
- Describe the set of all solutions (*parametrization*).
- Find a “perfect” estimator (*optimization* for secondary criterion)

Symmetry





Symmetry

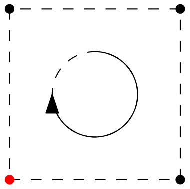


Symmetry

270°

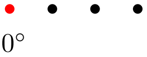
$90^\circ + 180^\circ$

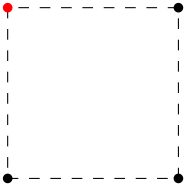
-90°



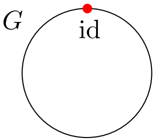
complete symmetry $\mathbb{Z}_4$

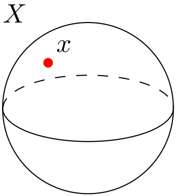
Symmetry





complete symmetry $\mathbb{Z}_4$





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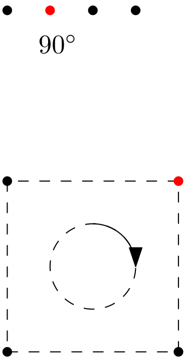

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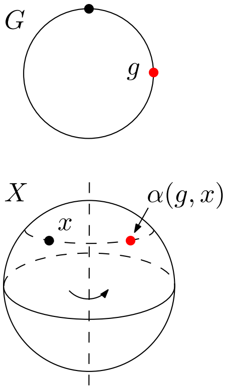
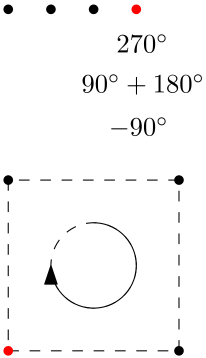
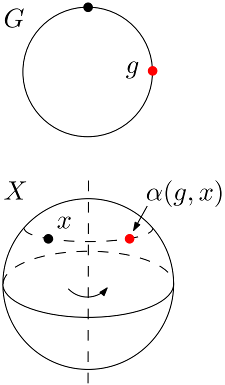


Symmetry



complete symmetry $\mathbb{Z}_4$



partial symmetry S^1
complete symmetry $\mathrm{SO}(3)$

Symmetry

Lie group G
differentiable manifold X

right action $\alpha\colon G \times X \to X$, $x \mapsto \alpha(g, x)$
$\alpha(\mathrm{id}, x) = x$ and $\alpha(g, \alpha(h, x)) = \alpha(hg, x)$

α transitive
$\Leftrightarrow$ X is a G-homogeneous space
$\Leftrightarrow$ G is a complete symmetry for X

Kinematic systems

Kinematic systems are of the form

$$\dot{x} = f(x, v),$$
$$y_i = h_i(x), \quad i = 1, \ldots, p$$

where $x(t) \in X$, a differentiable state manifold, $v(t) \in V$, an input vector space, and $f(x, .)\colon V \to T_x X$ *linear*.
Also, each $y_i(t) \in Y_i$, a differentiable output manifold.

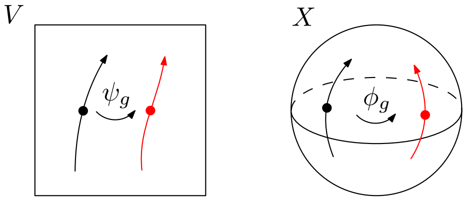
One way to think about kinematic systems is that they are defined by a *linearly* parametrized family $\{f(., v)\}_{v \in V}$ of vector fields on X.

Kinematic systems with complete symmetry

$$\dot{x} = f(x, v),$$
$$y_i = h_i(x), \quad i = 1, \ldots, p$$

with right Lie group actions

$$\phi \colon G \times X \to X,$$
$$\psi \colon G \times V \to V,$$
$$\rho_i \colon G \times Y_i \to Y_i$$



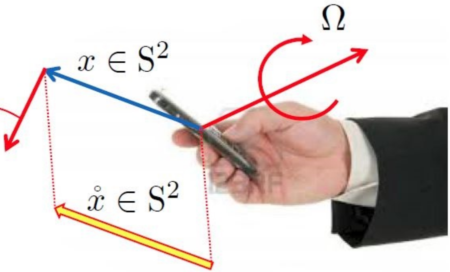
such that

$$\mathrm{d}\phi_g(x)[f(x, v)] = f(\phi(g, x), \psi(g, v)),$$
$$\rho_i(g, h_i(x)) = h_i(\phi(g, x))$$

A toy example

$$\dot{x} = x \times \Omega$$

$$y = x$$



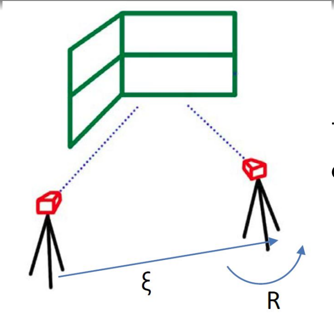
$$X = S^2,\ V = \mathbb{R}^3,\ Y = S^2, \qquad G = \mathrm{SO}(3)$$

$$\phi(R, x) = R^\top x$$

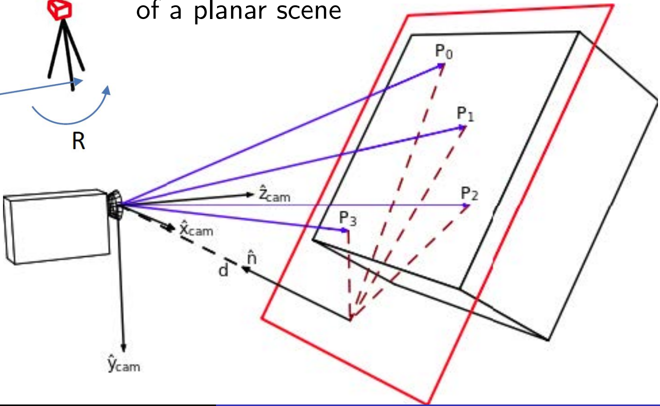
$$\psi(R, \Omega) = R^\top \Omega$$

$$\rho(R, y) = R^\top y$$

Homographies



Transformation of an image of a planar scene



$$H = R + \frac{\xi \hat{\eta}^\top}{d}$$

$$p_i \simeq H^{-1} \mathring{p}_i$$

Application of homographies to image stabilization

$$\dot{H} = H(\Omega_\times + \Gamma)$$
$$p_i = \frac{H^{-1}\mathring{p}_i}{\|H^{-1}\mathring{p}_i\|}$$

Ω is the angular velocity, Γ can be estimated concurrently with H, p_i can be obtained feature point correspondences in video frames

$$X = \mathrm{SL}(3),\ V = \mathfrak{sl}(3),\ Y_i = S^2, \qquad G = \mathrm{SL}(3)$$
$$\phi(Q, H) = HQ$$
$$\psi(Q, u) = Q^{-1}uQ$$
$$\rho_i(Q, p_i) = \frac{Q^{-1}p_i}{\|Q^{-1}p_i\|}$$

Robotics problems with symmetry

An incomplete list of robotics problems with complete symmetry:

- Attitude estimation $\mathrm{SO}(3)$
- Pose estimation $\mathrm{SE}(3)$
- Second order kinematics $\mathrm{TS?}(3)$
- Homography estimation $\mathrm{SL}(3)$
- Simultaneous Localization and Mapping (SLAM)
- Unicycle $\mathrm{SE}(2)$
- Nonholonomic car with trailers

These generic problems come in several versions depending on the types of available measurements.

General approach

- Lift the system kinematics to the symmetry group
- Design an observer for the resulting invariant system
- Project the observer state to obtain a system state estimate

Why?

- Observer design for invariant systems on Lie groups is very well studied (Bonnabel/Martin/Rouchon TAC 2009, Lageman/T./Mahony TAC 2010)
- It is often possible to obtain autonomous error dynamics in (global) gradient flow form
- The system theory of invariant systems on Lie groups is as close to LTI system theory as one can get in the nonlinear regime

Lifted kinematics

Fix a *reference point* $\boxed{\mathring{x} \in X}$ and choose a *velocity lift* $\boxed{F_{\mathring{x}} : V \to \mathfrak{g}}$ such that

$$\mathrm{d}\phi_{\mathring{x}}(\mathrm{id})[F_{\mathring{x}}(v)] = f(\mathring{x}, v)$$

Define *lifted kinematics*

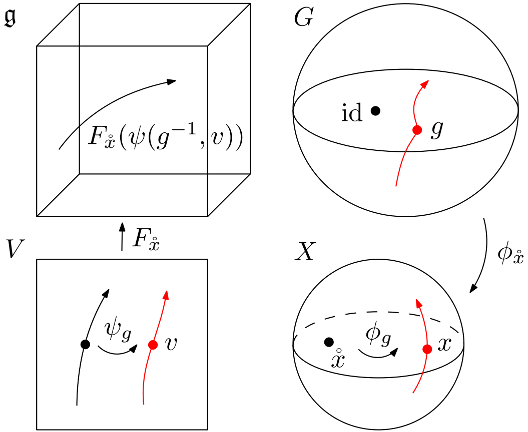
$$\boxed{\dot{g} = F(g, v) := \mathrm{d}R_g(\mathrm{id})[F_{\mathring{x}}(\psi(g^{-1}, v))], \quad y_i = \rho_i(g, \mathring{y}_i),}$$

where $\mathring{y}_i = h_i(\mathring{x})$, then

$$\mathrm{d}\phi_{\mathring{x}}(X)[F(g, v)] = f(x, v), \quad \text{where } x = \phi(g, \mathring{x})$$

The lifted kinematics on G project to the system kinematics on X!

Lifted kinematics

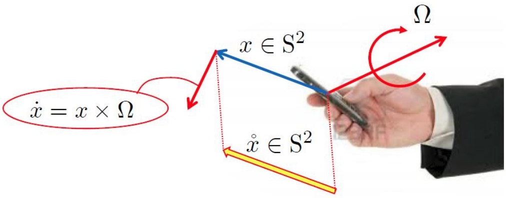


Symmetry R, ψ, ρ_i

Symmetry ϕ, ψ, ρ_i

Lifted kinematics

Toy example



$$F_{e_3}(\Omega) = \begin{pmatrix} 0 & -\Omega_3 & \Omega_2 \\ \Omega_3 & 0 & -\Omega_1 \\ -\Omega_2 & \Omega_1 & 0 \end{pmatrix} = \Omega_\times$$

$$F(R,\Omega) = (R\Omega)_\times R = (R\Omega_\times R^\top)R = R\Omega_\times$$

$\boxed{\dot{R} = R\Omega_\times}$ rigid body! $\boxed{\dot{H} = Hu}$ homography!

Type I lifted kinematics

A *right invariant* (physical) system description relative to an inertial frame with sensors attached to the body-fixed frame typically leads to *left invariant* kinematics on the symmetry group:

$$\dot{g} = \mathrm{d}R_g(\mathrm{id})[\mathrm{Ad}_g u] = \mathrm{d}L_g(\mathrm{id})[u], \quad u = F_{\mathring{x}}(v)$$

We call such systems *Type I*. A complete characterization of Type I (and Type II) symmetries has just been accepted for presentation at this year's CDC :-)

Type I systems allow particularly nice observer error dynamics.

Aside: Type II lifted kinematics

The seemingly more "natural" case of *Type II* lifted kinematics

$$\dot{g} = \mathrm{d}R_g(\mathrm{id})[u]$$

has been studied in the classical geometric control literature, see for example the work of Jurdjevic and Sussmann.

It turns out that this models the much rarer case of inertially based sensors that usually require "live" communication between the robot and a ground station (or a system such as GPS)!

Additionally, the error dynamics are not as simple as for Type I symmetries. For a detailed analysis of the attitude estimation problem in both cases see T./Mahony/Hamel/Lageman TAC 2012.

Observer design (for all types)

Lifted kinematics

$$\dot{g} = \mathrm{d}R_g(\mathrm{id})[F_{\mathring{x}}(\psi(g^{-1}, v))], \quad y_i = \rho_i(g, \mathring{y}_i),$$

Observer

$$\dot{\hat{g}} = \mathrm{d}R_{\hat{g}}(\mathrm{id})[F_{\mathring{x}}(\psi(\hat{g}^{-1}, v))] - \mathrm{d}R_{\hat{g}}(\mathrm{id})\Delta_{\mathring{y}}(\hat{g}, y), \quad \hat{g}(0) = \mathrm{id}$$
$$\hat{x} = \phi_{\mathring{x}}(\hat{g}),$$

where $\mathring{x}$ is chosen as the best guess of $x(0)$.

It remains to choose the *innovation term* $\Delta_{\mathring{y}}(\hat{g}, y)$ in a way such that $E_l := \hat{g}g^{-1} \to \mathrm{id}$.

Invariant innovation terms

An innovation term $\Delta_{\mathring{y}}(\hat{g}, y)$ is called *invariant* if

$$\Delta_{\mathring{y}}(\hat{g}h, \rho(h, y)) = \Delta_{\mathring{y}}(\hat{g}, y)$$

For an invariant innovation term and $y = \rho(g, \mathring{y})$,

$$\begin{aligned} \Delta_{\mathring{y}}(\hat{g}, y) = \Delta_{\mathring{y}}(\hat{g}, \rho(g, \mathring{y})) &= \Delta_{\mathring{y}}(\hat{g}g^{-1}g, \rho(g, \mathring{y})) \\ &= \Delta_{\mathring{y}}(\hat{g}g^{-1}, \mathring{y}), \end{aligned}$$

i.e.

$$\boxed{\Delta_{\mathring{y}}(\hat{g}, y) = \Delta_{\mathring{y}}(E_I, \mathring{y})}$$

Error dynamics - Type I with invariant innovation

Lifted kinematics

$$\boxed{\dot{g} = \mathrm{d}L_g(\mathrm{id})[F_{\mathring{x}}(v)]}$$

Observer

$$\boxed{\dot{\hat{g}} = \mathrm{d}L_{\hat{g}}(\mathrm{id})[F_{\mathring{x}}(v)] - \mathrm{d}R_{\hat{g}}(\mathrm{id})\Delta_{\mathring{y}}(\hat{g}, y)}$$

Error dynamics

$$\begin{aligned}
\dot{E}_I &= \frac{\mathrm{d}}{\mathrm{d}t}(\hat{g}g^{-1}) = \dot{\hat{g}}g^{-1} - \hat{g}(g^{-1}\dot{g}g^{-1}) \\
&= \mathrm{d}R_{g^{-1}}(\hat{g})\mathrm{d}L_{\hat{g}}(\mathrm{id})[F_{\mathring{x}}(v)] - \mathrm{d}R_{g^{-1}}(\hat{g})\mathrm{d}R_{\hat{g}}(\mathrm{id})\Delta_{\mathring{y}}(\hat{g}, y) \\
&\quad - \mathrm{d}L_{\hat{g}}(g^{-1})\mathrm{d}R_{g^{-1}}(\mathrm{id})[F_{\mathring{x}}(v)] \\
&= -\mathrm{d}R_{E_I}(\mathrm{id})\Delta_{\mathring{y}}(E_I, \mathring{y})
\end{aligned}$$

The error dynamics $\boxed{\dot{E}_I = -\mathrm{d}R_{E_I}(\mathrm{id})\Delta_{\mathring{y}}(E_I, \mathring{y})}$ are autonomous!

Constructing an invariant innovation term

Starting with individual smooth functions $f_i \colon Y_i \to \mathbb{R}^+$ with a global minimum at $\mathring{y}_i$, define the *aggregate cost*

$$\ell_{\mathring{y}}(\hat{g}, y) := \sum_{i=1}^{p} f_i(\rho_i(\hat{g}^{-1}, y_i))$$

The aggregate cost is *invariant*

$$\ell_{\mathring{y}}(\hat{g}, y) = \ell_{\mathring{y}}(E_I, \mathring{y})$$

and the right trivialization of its gradient w.r.t. a right invariant Riemannian metric

$$\Delta_{\mathring{y}}(\hat{g}, y) := \mathrm{d}R_{\hat{g}^{-1}}(\mathrm{id})\mathrm{grad}_1 \ell_{\mathring{y}}(\hat{g}, y)$$

is an invariant innovation term.

Observer for the toy example

System

$$\dot{x} = x \times \Omega, \quad \mathrm{y{=}x}$$

Lifted kinematics

$$\dot{R} = R\Omega_\times$$

Cost

$$f(y) = k\|y - \mathrm{e}_3\|_2^2$$

Observer

$$\dot{\hat{R}} = \hat{R}\Omega_\times - k(\mathrm{e}_3 \times \hat{R}y)_\times \hat{R}, \quad \hat{x} = \hat{R}\mathrm{e}_3$$

The main convergence result

Theorem (Mahony/T./Hamel, NOLCOS 2013)

Consider a kinematic system with a Type I complete symmetry. Assume that

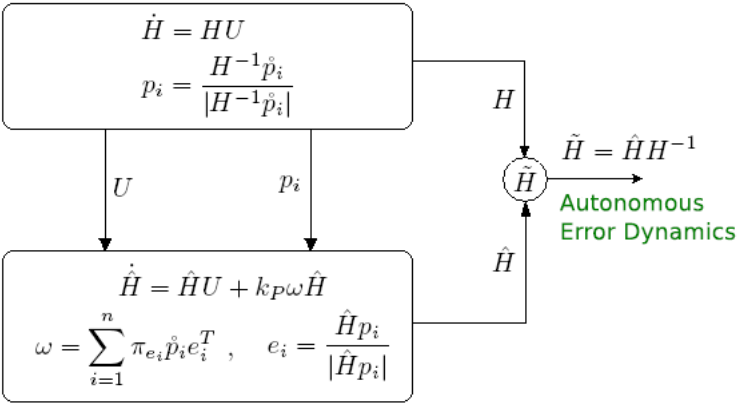
$$\bigcap_{i=1}^{p} \mathrm{stab}_{\rho_i}(\mathring{y}_i) = \{\mathrm{id}\}.$$

and construct an observer as above. Then

$$\dot{E}_I = -\mathrm{grad}_1 \ell_{\mathring{y}}(E_I, \mathring{y})$$

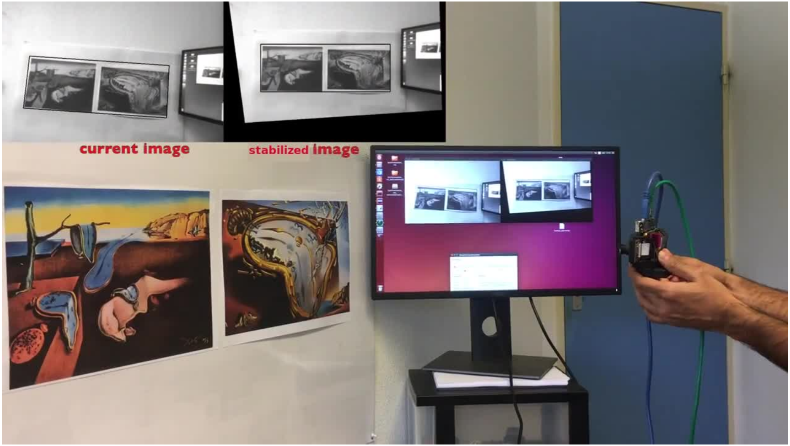
and $\hat{x}(t) \to x(t)$ at least locally, but typically almost globally.

A nonlinear homography observer



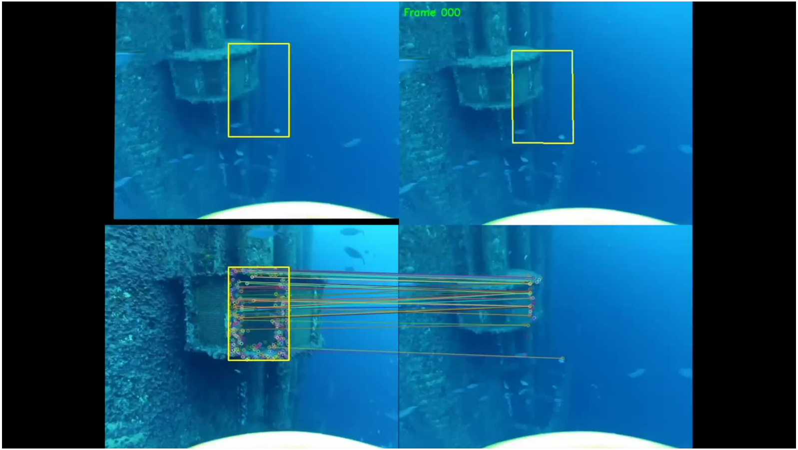
$$H \in \mathrm{SL}(3),\ U \in \mathfrak{sl}(3),\ p_i \in \mathrm{S}^2$$

Experimental results - Lab



credit: Minh Duc Hua (Laboratoire I3S, CNRS)

Experimental results - Underwater



credit: Minh Duc Hua (Laboratoire I3S, CNRS)

Outlook

- There are lots of open questions! Type II theory? Are there other types? Is there a general internal model principle?
- Extensions to biased input measurements, systems with measurement delays
- Minimum energy estimation or variational estimators (Sanyal et al.) as an alternative to nonlinear stochastic filtering
- Extensions to infinite dimensional systems
- Many essentially unexplored applications in robotics and computer vision

Unpaid advertisements

Invited tutorial session on "Geometric observers"

57th IEEE Conference on Decision and Control (CDC)
Miami Beach, FL, USA, December 17-19, 2018

Graduate course on "Nonlinear Observers: Applications to Aerial Robotic Systems"

Module M12, EECI International Graduate School on Control,
Genoa, Italy, April 8-12, 2019

Mohammad (Behzad) Zamani

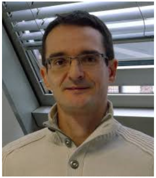
Alireza Khosravian

Christian Lageman

Minh Duc Hua

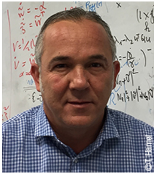
Alessandro Saccon

Thank you.



Pascal Morin

Pedro Aguiar



Tarek Hamel

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