

## THE AUSTRALIAN NATIONAL UNIVERSITY

# Modelling the Contact between a rolling sphere and compliant ground plane

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This thesis is submitted in part fulfilment of the degree of Bachelor of Engineering Department of Engineering Australia National University This thesis contains no material which has been accepted for the award of any other degree or diploma at any other university. To the best of the author's knowledge, it contains no materials previously published or written by another person, except where due reference is made in the text.

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## Abstract

Contact is a universally exist physical phenomenon. In previous work of Azad and Featherstone <sup>[8]</sup>, they present a complete non-linear 3D model which is capable of determining both normal and friction force. In this thesis their model and simulation results are discussed; an additional Stribeck model is incorporated to improve the model in low relative velocity region and therefore, supports an energy audit. An open loop experiment is also proposed aim to investigate the accuracy of the improved model.

## **Table of Contents**

Chapter 1 Introduction	9
1.1 Motivation of the Project	9
1.2 Background Information.	9
1.2.1 Characteristic of Contact	9
1.2.2. Classic models of Friction	10
1.2.3. Stiction.	11
1.2.4. Vicious Friction	11
1.2.5. Combination of the models	11
1.2.6. Stribeck Friction (or Stribeck effect)	12
1.2.7. Pre-sliding displacement (Dhal Effect)	12
1.3 Structure of the Thesis	13
Chapter 2 Literature Review.	14
2.1 Modelling Scenario	14
2. 2 Basic Formulas	15
2.2.1 Nonlinear Normal Force Model	15
2.2.2 Nonlinear Friction Force Model without Stribeck Effect	17
2.3 Matlab Simulation and Results	
2.3.1 Force simulation and results	19
2.3.2 Energy Audit	21
Chapter 3 Stribeck Effect Simulation	25
3.1 Phenomenon and Observations Related to Stribeck Effect	25
3.1.1 Dynamic Regimes and Steady state Friction vs. velocity Curve	e25
3.1.2 Stick-slip Phenomenon and friction lag	27
3.2 Stribeck Model selection	28
3.2.1 Models Justification	28
3.2.2 Model Selection	
3.2.3 Parameter justification	32
3.3 Matlab Simulation and Results	32

3.3.1 Friction Force Simulation and Results	32
3.3.2 Energy Audit	34
Chapter 4 Experiment Design	
4.1 Experiment aim	
4.2 Experiment Concept	
4.3 Experiment Tools and Materials Justification	37
4.3.1 Pulling Object Selection	37
4.3.2 Cork Plate	38
4.3.5 Back-of-the-Envelope Calculation and Justification	38
4.3.6 Pulling String	40
4.3.7 Force Transducer Selection	41
4.3.8 Data-acquisition systems	45
4.3.9 Linear Encoder	45
4.3.10 Driving mechanism	49
1 1 Experiment Procedure	52
4.4 Experiment Procedure	
Chapter 5 Conclusion.	
Chapter 5 Conclusion	56
<ul> <li>Chapter 5 Conclusion.</li> <li>5.1 Outcomes.</li> <li>5.2 Further Work.</li> </ul>	
Chapter 5 Conclusion. 5.1 Outcomes. 5.2 Further Work. Appendix	
Chapter 5 Conclusion. 5.1 Outcomes. 5.2 Further Work. Appendix A1.	
<ul> <li>4.4 Experiment Procedure</li> <li>Chapter 5 Conclusion.</li> <li>5.1 Outcomes.</li> <li>5.2 Further Work.</li> <li>Appendix</li> <li>A1.</li> <li>A2.</li> </ul>	
Chapter 5 Conclusion. 5.1 Outcomes. 5.2 Further Work. Appendix A1. A2. A3.	
Chapter 5 Conclusion. 5.1 Outcomes. 5.2 Further Work. Appendix A1. A2. A3. A4.	
4.4 Experiment Procedure         Chapter 5 Conclusion.         5.1 Outcomes.         5.2 Further Work.         Appendix         A1.         A2.         A3.         A4.         A5.	
4.4 Experiment Procedure         Chapter 5 Conclusion.         5.1 Outcomes.         5.2 Further Work.         Appendix         A1.         A2.         A3.         A4.         A5.         A6.	
4.4 Experiment Frocedure         Chapter 5 Conclusion.         5.1 Outcomes.         5.2 Further Work.         Appendix         A1.         A2.         A3.         A4.         A5.         A6.         A7.	
4.4 Experiment Procedure         Chapter 5 Conclusion.         5.1 Outcomes.         5.2 Further Work.         Appendix         A1.         A2.         A3.         A4.         A5.         A6.         A7.         A8.	
4.4 Experiment Procedure         Chapter 5 Conclusion.         5.1 Outcomes.         5.2 Further Work.         Appendix         A1         A2         A3         A4         A5         A6         A7         A8         A9	
4.4 Experiment Frocedure         Chapter 5 Conclusion.         5.1 Outcomes.         5.2 Further Work.         Appendix         A1.         A2.         A3.         A4.         A5.         A6.         A7.         A8.         A9.         A10.	

## List of Figures

Figure 1.1: Coulomb Friction Model11
Figure 1.2: Viscous Friction Model11
Figure 1.3: Examples of static friction models12
Figure 1.4: Friction vs. Displacement
Figure 1.5: Structure of thesis
Figure 2.1: Stress-strain curve of cork15
Figure 2.2: Side view of contact area16
Figure 2.3: Coulomb friction model
Figure 2.4: friction model with a variable-strength clutch
Figure 2.5: Normal force vs. time for a bouncing sphere
Figure 2.6: Enlargement for first bounce
Figure 2.7: Friction force in x direction vs. time
Figure 2.8: Friction vs. time for the first bounce
Figure 2.9: Velocity of steel ball in x-direction vs. time for the first bounce
Figure 2.10: Energy vs. time plot for a bouncing sphere
Figure 2.11: Energy vs. time plot for a rolling sphere
Figure 2.12: Total energy difference vs. time (without clutch design)
Figure 2.13: Total energy different vs. time (with clutch design)24
Figure 3.1: The generalized Stribeck curve, showing friction as a function of velocity
for low velocities
Figure 3.2: Friction as a function of steady state velocity for various lubricants25
Figure 3.3: The friction-velocity relation observed by Hess and Soom
Figure 3.4: Friction models: curve
Figure 3.5&3.6: Bo and Pavelescu's model for $\dot{x}_s = 0.1 \sigma = 2 \& 1$ respectively30
Figure 3.7&3.8: Bo and Pavelescu's model for $\sigma$ =0.1 $\dot{x}_s$ =0.1 & 0.02 respectively30
Figure 3.9: Lorentzian model for Stribeck velocity set to be 0.1
Figure 3.10: Enlargement of the Lorentzian model in the boundary and partial fluid
lubrication region

Figure 3.11&3.12: Friction coefficient vs. clutch velocity	33
Figure 3.13&3.14: Enlargement of Friction coefficient vs. clutch velocity	33
Figure 3.15&3.16: Friction force in x direction vs. time and Friction vs. time for	the
first	

bounce	34
Figure 3.17: Total energy different vs. time (with Stribeck effect)	34
Figure 3.18: Total energy different vs. time	35
Figure 4.1: Function of Velocity at Low Velocities, cure given by Tustin's Expone	ential
Model	39
Figure 4.2: Friction as a Function of Velocity during Motions against a Compla	inant
Surface	39
Figure 4.3: Simplified RC circuit of Piezoelectric Force Sensor	43
Figure 4.4: Discharge Time Constant (TC) Output vs. Time	43
Figure 4.5: gate voltage discharge pattern up to 1% TC	44

Figure	4.6&	Figure	4.7:	Image	Scanning	Principle	and	Interferential	scanning
princip	le					•••••			47
Figure	4.8&4	.9: Posi	tion e	error ov	er the mea	suring len	gth N	IL and Position	on error u
within	one sig	nal peri	od					•••••••••••••••••••••••••••••••••••••••	49
Figure	4.10: S	peed an	d forc	e chara	cter diagrai	n			

## **Chapter 1 Introduction**

#### **1.1 Motivation of the Project**

Generally, there are two force components in a contact, which are tangential friction and normal force. To develop a comprehensive understanding of the contact, there must be an understanding of its components. Friction is emphasized in this project, it is sophisticated in modelling as there is a wide range of physical phenomena relate to it, like friction lag, Stribeck effect, pre-slide displacement etc. Numerous experiments and simulation have been conducted to capture and study these friction-related phenomena, apart from intellectual curiosity, understanding to friction is driven by strong engineering and industry needs. However, due to the high irregularity of the contact surface and material property diversity, a mathematical form which is able to cover all friction phenomena and describe friction accurately is not yet available. Understand the present friction models and its applied conditions can help to improve and refine the present contact model in both forces accounting and energy audit.

#### **1.2 Background Information**

#### **1.2.1 Characteristic of Contact**

Contact is a complex physical phenomenon, which occurs when two or more bodies collide with each other. There are four major characteristics of contact, which are:

- 1. Very short duration
- 2. High force levels reached
- 3. Rapid dissipation of energy
- 4. Large accelerations and decelerations present<sup>[1]</sup>

These characteristics make the essential phenomenon hard to capture, so it is a good idea to break down the phenomenon in to components and analyse them one by one. As we mentioned before, the two major components in contact are friction and normal force. In order to model the contact precisely, it is desirable for a model to capture both forces.

Firstly, for modelling the normal force, there are two general types, compliant and rigid models, which are also known as continuous and discontinuous approach respectively. The continuous contact model assumes the forces and deformation change in a continuous manner when a collision takes place <sup>[2].</sup> In the discontinuous method it is assumed that the impact occurs instantaneously and the integration of the equations of motion is halted at the time of impact. In the discontinuous method, the dynamic analysis of the system is divided into two intervals, before and after impact. This method is commonly referred to be a piecewise analysis, and has been used for solving the intermittent motion problem <sup>[2]</sup>.

Secondly, friction force and friction phenomena are also discussed in this paper. Friction is a complicated non-linear physical phenomenon, which appears at the physical interface between two surfaces in contact. Friction occurs in all mechanical systems, and friction models can be classified as static and dynamic <sup>[3]</sup>, via different presentation of equations. Static model is classified as a function of relative velocity; dynamic model is defined as a function of relative displacement and velocity between two objects. Before we go deep into models and simulation sections, five essential friction effects and friction phenomena are discussed.

#### **1.2.2.** Classic models of Friction.

According to the explanation of Leonardo Da Vinci, 'friction is proportional to load, opposes the direction of motion and is independent of contact area'. Guillaume Amontons rediscovered Da Vinci's model and the refinement is done by Charles Augustin Coulomb <sup>[4]</sup>, the refined model now is known as Coulomb friction model (figure 1.1). It can be interpreted shown in Eq.1.1, where  $\mu$  is the dynamic/coulomb friction coefficient and F<sub>n</sub> is the normal force.

$$F=\mu_c \times F_n, \qquad (1.1)$$



Figure 1.1: Coulomb Friction Model<sup>[5]</sup>

#### 1.2.3. Stiction

Stiction is the static friction which needs to be overcome to enable relative motion of stationary objects in contact. For two bodies in physical contact, stiction force at rest is greater than Coulomb friction level. The maximum stiction can be expressed in equation 1.2, where  $\mu_s$  is the static friction coefficient.

$$F = \mu_s \times F_n \tag{1.2}$$

#### **1.2.4. Vicious Friction**

In the  $19^{\text{th}}$  century, with the development of hydrodynamics, Renolds (1866) developed expressions for the friction force caused by the viscosity of lubricates (Eq.1.3) <sup>[6]</sup>, which is normally described as the multiplication of viscous friction coefficient and velocity (figure 1.2).

$$F = \mu_v \times v \tag{3}$$



Figure 1.2: Viscous Friction Model<sup>[5]</sup>

#### 1.2.5. Combination of the models

The classical friction components can be combined in different ways, shown in Figure 1.3. Figure 1.3a) shows Coulomb friction; Figure 1.3b) combines Coulomb friction and viscous friction; Stiction plus Coulomb and viscous friction is shown in Figure 1.3c) and Figure 1.3d) shows how the friction force may decrease continuously from the static friction level.



Figure 1.3: Examples of static friction models <sup>[6]</sup>

#### 1.2.6. Stribeck Friction (or Stribeck effect)

Stribeck (1902) observed that <sup>[4]</sup>, when the relative velocity between two objects is low, the friction force is decreasing continuously with increasing velocities and not in a discontinuous matter as described in figure 1.3c. This phenomenon of a decreasing friction at low, increasing velocities is called the Stribeck friction or effect (shown is figure 1.3d).

#### **1.2.7.** Pre-sliding displacement (Dhal Effect)

The pre-sliding displacement is a dynamic friction Phenomena. Dhal observed that <sup>[7]</sup>, when two objects come in contact and the external force is smaller than the maximum stiction force, there will be elastic deformation along the tangential direction <sup>[3]</sup>. That is because surfaces are very irregular at the microscopic level and two surfaces therefore make contact at a number of elastic 'asperities', which may deflect like a spring. When the applied force is released, the displacement will go

back as shown in Figure 1.4. Due to the high irregularity of these asperities, if the force is sufficiently large, some of the asperities will slip and result in permanent displacement.



Figure 1.4: Friction vs. Displacement <sup>[6]</sup>

#### **1.3 Structure of the Thesis**

The flow diagram show in Figure 1.5 illustrates the structure of the thesis. Project purpose and fundamental understanding of friction models and related phenomena are covered in Chapter 1. Azad and Featherstone's <sup>[8]</sup> contact model as well as their simulation results are mainly reviewed in chapter 2. In chapter 3, models which are able to capture friction changes in low velocity region are discussed and compared. Afterwards, simulations based on the Stribeck model selected are performed. Simulation results are discussed at the end of the chapter 3. Chapter 4 justifies the experiment design, apparatus and procedures. In the final chapter, conclusions are conducted, which includes the outcome of the model as well as the future plan and works to improve the project.





### **Chapter 2 Literature Review**

#### 2.1 Modelling Scenario

In this chapter, Azad and Featherstone's pervious work: modelling the contact between a rolling sphere and a complaint ground <sup>[8]</sup> will be discussed. In this modelling, steel sphere with a mass of 0.154kg and cork plate are chosen, it describes the scenario, a steel ball with a initial tangential velocity of  $\sqrt{0.5}$ m/s drops from a height of 0.1m above the plate (where the plate is parallel to the ground).

Chapter 1 mentioned that, there are two general types of modelling for the normal force: rigid and compliant. For rigid model, the discrete formulation is based on the assumptions that, the impact process is instantaneous; kinetic variables have discontinuous changes while no displacements occur during the impact, and other finite forces are negligible <sup>[9]</sup>, which indicates that, this model is used mainly if the impact involves rigid or very hard compact bodies. According to the Stress-strain curve of cork (Figure 2.1), the stiffness module of cork is quite low, a significant deformation (about  $1 \times 10^{-4}$ m) on the surface of cork plate is expected on impact with the steel ball drop from 0.1m above the plate. In the consideration of energy audit, the ball will experience considerable amount of energy dissipating continuously during each impact, due to the deformation of the impact area. Hence the rigid model is not suitable in this scenario.

Compliant models do not have such issues, but before the problem in elasticity can be formulated, a description of the geometry of contacting surfaces is necessary. According to Hertz's theory <sup>[11]</sup>, each surface in contact is needed to be considered topographically smooth on both micro and macro scale. On the micro scale, this implies: the absence or disregard of small surface irregularities which would lead to discontinuous contact or highly local variations in the contact pressure. On the macro scale, the profiles of the surfaces should be continuous up to their second derivative in the contact region <sup>[11]</sup>. In this case here, sphere steel ball and cork plate with smooth surface meet the geometry requirement for this non-rigid modelling. To conclude, after we discussed the mechanical properties and shape of the two objects, compliant method is suitable in this modelling scenario.



Figure 2.1: Stress-strain curve of cork<sup>[10]</sup>

#### 2. 2 Basic Formulas

#### 2.2.1 Nonlinear Normal Force Model

In Azad and Featherstone's normal force model <sup>[8]</sup> there are two terms: spring term and damper term, this allows capturing the phenomenon of bouncing and energy dissipation in contact. Their model is developed from Hertz's model. Eq. 2.1 shows a basic Hertz's model <sup>[9]</sup>. This model is limited to impacts with elastic deformation, and excludes the consideration of energy dissipating. Use this model; the contact process can be pictured as two bodies interacting via a non-linear spring along the line of impact. The hypotheses used states that the deformation is concentrated in the vicinity of the contact area,

$$\mathbf{F}_{n} = \mathbf{K}\mathbf{z}^{n} \tag{2.1}$$

In the Hertz's equation, where z is the deformation of the ground, K is coefficient of spring and n is a constant, depending on material and geometric properties and can be computed by using elastostatic theory. In the case here, a steel sphere impact with a cork plate, n is set to be 3/2 and k is defined in terms of the two contact surfaces'

Poisson's ratios ( $v_1$ ,  $v_2$ ), Young's moduli ( $E_1$  and  $E_2$ ) and the radius of the sphere (r), due to Hertz's theory <sup>[12]</sup>. If the ground is considered to contain a uniform distribution of infinitely many non-linear spring structures, the ground can be characterized by stiffness per unit area. So the normal force due to spring is given by Eq. 2.2.

$$f_{\rm K} = \int_0^z A(\xi) \ K_{\rm A} (z - \xi) d\xi$$
 (2.2)

Where  $K_A(z)$  The coefficient of stiffness per unit area and should be chosen as:

$$K_{A}(z) = \frac{E^{*}}{2\pi\sqrt{r}} z^{\frac{-1}{2}}$$
(2.3)

And E\* is determined by:

$$E^* = \frac{1 - \nu_1^2}{E_1} + \frac{1 - \nu_2^2}{E_1}$$
(2.4)

A(z) is the area of contact expressed as function of z,  $\xi(A)$  denotes the local deformation at the area element dA. Figure 2.2 illustrates the contact area is calculated for a sphere of radius r. Azad and Featherstone <sup>[8]</sup> define the contact area to be the area undeformed ground that makes contact with the sphere, So we have:



Figure 2.2: Side view of contact area<sup>[8]</sup>

Assuming that  $z \ll 2r$  and substitute Eq. 2.3 and 2.5 into equation 2.2. An equation for  $z \ge 0$  is obtained:

$$f_{\rm K} = \frac{4}{3} E^* \sqrt{r} * z^{\frac{3}{2}}$$
(2.6)

Above is the friction force due to the spring, now let's consider another model, which first proposed by Hunt and Crossley<sup>[13]</sup>:

$$F = kz^{n} + \lambda z^{p} \dot{z}^{q}$$
(2.7)

Where it is standard to set  $p = n = \frac{3}{2}$  and q = 1 <sup>[14], [15], [16]</sup>. In the spring-damper model,

the damping parameter  $\lambda$  can be related to the coefficient of restitution, since both are related to the energy dissipated by the impact process. In similar contact modelling cases, Hunt and Crossley <sup>[14]</sup>, Lankarani and Nikravesh <sup>[15]</sup>, and Marhefka and Orin <sup>[16]</sup> established:

$$e = 1 - \alpha \dot{z}_0, \quad \alpha = \frac{2\lambda}{3k}$$
 (2.8)

Where k is coefficient of spring, and  $\lambda$  is coefficient of restitution.

Based on the model instead of considering the ground contain only strings, now the ground contains a uniform distribution of infinitely many non-linear spring-damper pairs. According to Azad and Featherstone <sup>[8]</sup> the normal force due to friction is shown in equation 2.9:

$$f_{\rm D} = \int_{A(z)} D_A(\xi(A)) \dot{z} \, dA \tag{2.9}$$

Follow the similar procedure that  $f_K$  is derived we have:

$$D_A(z) = \alpha z^{\frac{-1}{2}}, f_D = 4\pi r \alpha z^{\frac{1}{2}} \dot{z}$$
 (2.10)

Further simplify the  $D_A(z)$  and  $K_A(z)$  by setting them equal to Dn and Kn respectively, where Dn and Kn are spring and damping coefficient that depends only on mechanical properties of the contacting surface. Combine  $f_D$  and  $f_K$  the total normal force is obtained:

$$F = D_n z^{\frac{1}{2}} \dot{z} + K_n z^{\frac{3}{2}}$$
(2.11)

#### 2.2.2 Nonlinear Friction Force Model without Stribeck Effect

In this section, a model describes the friction force during the contact period will be discussed. The model is essentially, a nonlinear, 2D version of the friction model introduced by Featherstone in his previous paper <sup>[17]</sup>. A physical interpretation of the model is shown in Figure 2.3. In the figure, the spring and damper pair on the left is the normal force model we discussed before, the spring and damper pair on the right is the friction force model. This additional compliance can be used to implement a Coulomb friction model in which the friction force is a function of only the position and velocity variables. There are two new variables introduced by the friction model:

u and  $V_{sphb}$  u is the displacement in the tangent direction of the surface patch from its equilibrium position; while  $V_{sph}$  is the tangential velocity of the point in B that is currently in contact with S.



Figure 2.3: Coulomb friction model<sup>[17]</sup>

A mathematical interpretation of this friction model can be expressed in Eq. 12

$$f_{stick} = -k_t u - b_t V_{sph}$$
(2.12)

Where  $k_t$  and  $b_t$  are stiffness and damping coefficients respectively and they are functions of the contact area (A(z)), and the tangential stiffness and damping coefficients per unit area (KA(z) and DA(z)). Similar to the normal spring-damper case, assume the contacting surfaces are isotropic, following the same calculation procedure the stiction force can be written as:

$$f_{\text{stick}} = -K_t z^{\frac{1}{2}} u - D_t z^{\frac{1}{2}} V_{\text{sph}}$$
(2.13)

Where:

$$K_t = 2E^* \sqrt{r} \tag{2.14}$$

$$D_t = 4\pi r \alpha \tag{2.15}$$

According to the Coulomb friction model, force can now be expressed as follows:

$$F_{f} = \begin{cases} -\mu F_{n} \text{ if } f_{\text{stick}} < -\mu F_{n} \\ \mu F_{n} \text{ if } f_{\text{stick}} > \mu F_{n} \\ f_{\text{stick}} \text{ otherwise,} \end{cases}$$
(2.16)

where  $\mu$  is the coefficient of friction. In words, if ft lies inside the friction cone,  $F_f$  is constrained to lie in the friction cone. If there is a force in this range that can prevent slippage between B and S, then the  $F_f$  value is equal to  $f_{stick}$ ; otherwise, ft lies on the nearest range boundary, and some slippage (not significant amount) does occur.

An additional clutch model can be added and it is designed to slip when the ground reaction force reaches the edge of the friction cone. The physical interpretation is shown in figure 2.4.



Figure 2.4: friction model with a variable-strength clutch <sup>[8]</sup>

The combination of the normal force model and the friction force model (with clutch design), describes the situation: on the impact of the steel ball and the plate, the point on impact and its vicinity area will deform due to the mass of the ball, in other words, the plate on impact will give away its original position, at a rate of  $\dot{u}$ . When the ball and plate are in contact, if the tangential velocity at the bottom point of the ball is greater than the deformation rate  $\dot{u}$ , then slip occurs. The friction force in the slip region can be expressed as:

$$f_{slip} = f_{stick} * \frac{\mu F_n}{|f_{stick}|} - C_v V_{clutch}$$
(2.17)

In this equation,  $C_v$  is the viscous friction parameter and  $V_{clutch}$  is the slipping velocity between the sphere and the ground.

#### 2.3 Matlab Simulation and Results

#### 2.3.1 Force simulation and results

In this Matlab simulation, the normal force coefficients Kn and Dn are set to be  $8.5 \times 10^{6}$ Nm<sup>-1</sup> and  $3.1 \times 10^{6}$ Nms<sup>-1</sup> accordingly; the friction force coefficient Kt and Dn are equal to  $12.75 \times 10^{6}$ Nm<sup>-1</sup> and  $3.1 \times 10^{6}$ Nms<sup>-1</sup> respectively; 0.2 is chosen for the friction coefficient.



Figure 2.5&2.6: Normal force vs. time for a bouncing sphere (left) and Enlargement for first bounce (right)

Figure 2.5 shows the normal force vs. time bouncing. As can be seen from the diagram, normal force experiences a successive decline over time. In the first bounce, the normal force can reach as large as 240N, but after few bounces the normal force decline to 0. In figure 2.5, for each major contact, there seems to be overlaps in vertical lines, as we zoom in the first bounce (figure 2.6), a closed curve is obtained. This curve is reasonable as the normal force is increasing initially as the ball 'get more and more into the ground'. Later the force become smaller is a result of spring releasing its stored energy.



Figure 2.7&2.8: Friction force in x direction vs. time (left) and Friction vs. time for the first bounce (right)

Figure 2.7 shows the pattern of friction force over time, different form the normal force case, the ball experiences friction direction changes in a single contact. As we

zoom in the first contact (figure 2.8), we find that, the ball experiences a negative friction force first and later the direction friction changes gradually from negative to positive (Negative friction is because friction is a force goes against motion and the initial ball's velocity is set to be positive). This indicates that, at the initial stage of the contact, parts of the ball's kinetic energy is stored in the 'spring', later, energy in 'spring' are released and 'help' the ball to move forward. This explanation is support by figure 2.9 as the velocity of ball in x direction drop to around 0.25m/s during the first impact and later the value restored to roughly 0.3m/s. The friction force in the y direction vs. time diagram has the same pattern as friction force in x direction case, this is because the contacting surface is isotropic and the initial velocity in x and y are the same, the expected magnitude of friction in x and y direct are same. These diagrams are from simulation with the additional clutch design, compare with the simulation diagrams without clutch design <sup>[8]</sup>, significant difference cannot be observed in force simulation.



Figure 2.9: Velocity of steel ball in x-direction vs. time for the first bounce

#### 2.3.2 Energy Audit

According to the law of conservation of energy, the total amount of energy in an isolated, pure mechanical system should remain constant over time, which means the sum of energy stored in the spring, energy dissipated by damper, total potential energy and kinetic energy of the system should be a constant value.

The only dissipative component in the model is the damper and the amount of

energy dissipated is:

$$E_{\text{Damper}} = E_{\text{NDamper}} + E_{\text{TDamper}} = \int_0^t f_D \dot{z} dt + \int_0^t D_t z^{\frac{1}{2}} \dot{u}^2 dt \qquad (2.18)$$

The energy stored in the normal and tangential spring can be calculated as:

$$E_{Sp} = E_{NSp} + E_{TSp} = \int_0^t K_n z^{\frac{3}{2}} \dot{z} dt + \int_0^t K_t z^{\frac{1}{2}} u \dot{u} dt$$
(2.19)

When the friction which the sphere experiences is inside the 'sticking regime', total energy dissipated in spring and damper can be calculated via Eq. (18), however, if the sphere is 'slipping', the energy dissipation due to clutch is presented in Eq. (20):

$$E_{clutch} = \int_0^t F_f V_{clutch} dt \qquad (2.20)$$

Where V<sub>clutch</sub> is determined by:

$$V_{clutch} = V_{sph} - \dot{u} \tag{21}$$

The major part in the system which experiences kinetic and potential energy change is the steel ball, and the energy of the ball can be expressed as:

$$E_{\text{ball}} = \text{mgh} + \frac{1}{2}\text{m}\dot{c}^2 \tag{22}$$

Figure 2.10&2.11 shows the total energy vs. time plot for bouncing sphere and rolling sphere respectively <sup>[8]</sup>, in the two figures, as can be seen, the amount of damped and stored energy are recorded at all simulation time. The damped energy gains after each contact while at the same time the stored energy decrease. Total energy of the system remains almost constant in both figures.



Figure 2.10: Energy vs. time plot for a bouncing sphere <sup>[8]</sup>



Figure 2.11: Energy vs. time plot for a rolling sphere <sup>[8]</sup>

In order to see the energy conservation condition in a detail level, we can use the initial energy to be subtracted by total energy of the system for all simulation time. The initial energy is calculated by Eq. (22), since there is no contact take place at the starting point.

Figure 2.12 shows Total energy difference vs. time plot which excluded the clutch design; it has an overall negative value which suggests the whole system gains energy after impacts. After 0.6 second the total energy stabilized at the level around  $-2.8 \times 10^{-5}$  N·m. For Figure 2.13, the total energy difference does have a negative overall value, but the clutch design does make the energy difference gap less significant, and the plot flattens out at a much lower level around  $-5 \times 10^{-6}$ N·m. From the results, we can conclude that the addition clutch term in the friction model gives a more accurate simulation in energy audit, but why the overall energy difference still a negative value? As mention previously, this simulation only takes the major part (steel ball) in the system which experiences kinetic and potential energy change into consideration and excluded the plate. To make a more accurate simulation, we can either take the plate into the kinetic and potential energy change calculation, or we can refine the current model by accounting the Stribeck effect into consideration. This paper in mainly focused on the modelling of contact thus, Stribeck model option is selected and discussed in chapter 3.



Figure 2.12: Total energy difference vs. time (without clutch design)



Figure 2.13: Total energy different vs. time (with clutch design)

### **Chapter 3 Stribeck Effect Simulation**

In chapter 2, Azad and Featherstone's Friction model<sup>[8]</sup> are discussed. Their model covers coulomb friction and viscous friction but excluded friction phenomena in low velocity region. In order make the fiction model more accurate; model which is able to capture Stribeck effects is incorporated into the present model. Before we go deep into the model selection phase, phenomena and friction-velocity curve pattern related to Stribeck model is introduced. Secondly, Several Stribeck models proposed by other researchers are explained. Afterwards, an appropriate Stribeck model is selected and reason of choice is justified. Finally, a friction model combines Stribeck effect is simulated and the results are discussed.

#### **3.1 Phenomenon and Observations Related to Stribeck Effect**



#### 3.1.1 Dynamic Regimes and Steady state Friction vs. velocity Curve

Figure 3.1: The generalized Stribeck curve, showing friction as a function of velocity for low velocities <sup>[18]</sup>

Figure 3.2: Friction as a function of steady state velocity for various lubricants <sup>[18]</sup>

According to experiment data and empirical results of Stribeck and Czichos <sup>[30], [20]</sup>, in a real scenario for system with lubricates, there are four regimes of lubrication in a system which are: static friction, boundary lubrication, partial fluid lubrication and full fluid lubrication (figure 3.1). The characteristic of friction in these velocity regions are listed below:

Regime 1: it is often assumed that there is no motion in the regime and force is proportional to displacement (due to spring deformation) which is governed by Eq.2.13. When the distance beyond the 'break distance' -on the order of 5 microns for steel junction <sup>[21]</sup>-, the Dahl effect may be minute.

Regime 2: in the regime, friction is largely independent of velocity and strongly dependent on lubricant chemistry <sup>[18]</sup>.

Regime 3: friction decreases with increasing velocity can be observed if static friction is greater than kinetic friction. According to Armstrong <sup>[18]</sup>, friction observed in the region may fluctuates, but by proper choice of lubricant the instability can be reduced or eliminated

Regime 4: friction is a function of velocity in this regime; a viscous plus kinetic friction model (Eq. 2.17) may describe the friction.

Figure 3.2 presents several friction-velocity Curves, which are depended upon the degree of boundary lubrication and details of partial fluid lubrication.

Curve (a): it arises when very limited boundary lubrication is provided by the applied lubricants. The data from Hess and Soom <sup>[22]</sup> indicate the plain oil gives such a curve. Curve (b): if lubricates are effective in boundary lubrication regime, the friction is relatively constant up to the regime where partial fluid lubrication starts to function <sup>[23]</sup>. According to Fuller <sup>[24]</sup>, objects running in steady state velocity, lubricated by plain oil with lubricity additive give a cure of type b. One must be careful in discussing curve types, because effects of frictional lag plays a significant role during velocity transients and data collected may exhibit a curve of type (b) even if the actual type is (a) <sup>[18]</sup>.

Curve (c): The boundary lubrication provided by way lubricants give a curve of type c <sup>[25]</sup> and it reduces static friction to a place below the coulomb friction level.

#### 3.1.2 Stick-slip Phenomenon and friction lag

#### **Stick-slip Phenomenon**

The explanation of Stick-slip phenomenon is based on the understanding of pre-sliding displacement, when this external force beyond the maximum force which could be stored in these elastic asperities, relative motion between two bodies occurs. 'If relative motion is accompanied by a reduction in the friction force, the sliding body will accelerate until the point where the elastic restoring force and the friction force between the sliding bodies equalize and deceleration takes place until a new period of stick occurs' according to Bowden and Tabor <sup>[26]</sup>. Stick-slip phenomenon can be observed in unsteady motion conditions and it can be easily identified in the first three dynamic regimes <sup>[27]</sup>. In addition, the stick-slip phenomenon can be eliminated by stiffening a mechanism <sup>[18]</sup>.

#### **Frictional lag**

Frictional lag or hysteresis effect observed around the Stribeck velocity region is shown in Figure 3.3. From the figure, the friction force for increasing velocities is larger than the friction force for decreasing velocities, thus the magnitude of friction force depends on the state of motion. Due to experiment results from Hess and Soom, the size of the hysteresis loops varies with normal load, viscosity and frequency of the velocity variation <sup>[6]</sup>.



Figure 3.3: The friction-velocity relation observed by Hess and Soom<sup>[6]</sup>

#### 3.2 Stribeck Model selection

#### **3.2.1 Models Justification**

Knowing the pattern of Stribeck curves with various degree of boundary lubrication, to simulate the Stribeck effect, it is important to have a mathematical model of friction-velocity. For the moment, no predictive mathematical form for the Stribeck friction is available, thus we are going to discuss some models which are based on empirical results and select one appropriate model for simulation.

In 1969, Bell and Burdekin<sup>[28]</sup> proposed a linearised discontinuous friction model, the friction force according to the model, linearly decreasing with slip velocity from static value to the slip acceleration phase (Figure 3.4, model 2). Cockerham and Symmons improved this model by developed a stability relationship.

Much unlike those linear models above, Bannerjee proposed a continuous second-order polynomial relation between the friction force and slip velocity based on the assumption that  $F_0 = F_k$  (figure 3.4, model 5, Eq.3.1).

$$F(V_r) = F_s - \alpha V_r + \beta V_r^2$$
(3.1)

Where:

$$V_{\rm r} = (\dot{x}/\dot{x}_{\rm s}) \tag{3.2}$$

$$\alpha = \gamma(Fs - Fkmin) \tag{3.3}$$

$$\beta = \frac{1}{2}\gamma^2 (Fs - Fkmin)$$
(3.4)

#### $\gamma$ is a constant

Based on the results of an analogue simulation regarding stick-slip stability conditions, two linear friction models (Figure 3.4, models 3 and 4) proposed by Cockerham and Cole illustrate the existence of three different conditions of stability in stick-slip motion <sup>[27]</sup>. Bo and Pavelescu confirmed the conclusion of Cockerham and Cole via experiment, they observe that:

1. When the relative velocity is low, friction force is decreases with increasing relative velocity and friction drops from static friction level Fs to a lower value  $F_k$ .

2. When the relative velocity is low, friction force increases with decreasing relative velocity and when the relative velocity becomes equal to zero the friction force

reaches a certain value F<sub>0</sub>.

From the observation, state of motion plays an essential role in determining the friction force. Two models are proposed by Bo and Pavelescu due to the presence of frictional lag:

$$F(\dot{x}) = F_k + (F_s - F_k)e^{-(\dot{x}/\dot{x}_s)^{\sigma}}$$
(3.5)

$$F(\dot{x}) = F_k + (F_0 - F_k) e^{-(\dot{x}/\dot{x}_s)^{o}}$$
(3.6)

For a particular experiment set ups, cast iron sliding on cast iron and steel sliding on steel surface, Bo and Pavelescu find the constant  $\sigma$  ranges between  $\frac{1}{2}$  and  $1^{[27]}$ . A viscous term suggested by Armstrong <sup>[18]</sup> is incorporated into the exponential model, and the model becomes:

$$F(\dot{x}) = F_k + (F_s - F_k)e^{-(\dot{x}/\dot{x}_s)\sigma} + F_v \dot{x}$$
(3.7)

$$F(\dot{x}) = F_k + (F_0 - F_k) e^{-(\dot{x}/\dot{x}_s)^{\sigma}} + F_v \dot{x}$$
(3.8)



Figure 3.4: Friction models: curve 1, Bowden and Blok model; curve 2, Bell and Burdekin model; curves 3 and 4, Cockerham and Cafe models; curve 5, Bannerjee model; curve 6, model of the Bo and Pavelescu<sup>[27]</sup>.

However, Bo and Pavelescu's model is not a strong constrain, curve type (a), (b), (c) shown is figure 3.2 all can be obtained by selecting appropriate parameter<sup>[18]</sup>. According to simulation results, by changing the number of  $\sigma$ , a constant friction vs. velocity pattern can be observed in boundary lubrication regime (Fig.3.5&3.6). The

magnitude of Stribeck velocity ( $\dot{x}_s$ ) determines how fast the friction drop in the partial fluid lubrication region (Figure 3.7&3.8). In addition, the curve type (c) can be obtained via changing the sign of Stribeck velocity.



Figure 3.5&3.6: Bo and Pavelescu's model for  $\dot{x}_s = 0.1 \sigma = 2 \& 1$  respectively



Figure 3.7&3.8: Bo and Pavelescu's model for  $\sigma=0.1$   $\dot{x}_s=0.1$  & 0.02 respectively

Starts with a well know Stribeck diagram <sup>[29]</sup>, Hess and Soom <sup>[22]</sup> set an experiment and investigated the systematic relationship between sliding velocities, loads and lubricate viscosity. In their experiment, a disk is designed to form a linear contact with a circular flat rider button (shown in A1) and the operation is to be carried out under unsteady sliding velocity. Contact materials are made of 52100 steel with a Brinell hardness of 192 <sup>[22]</sup>. According to this close loop experiment design, Lorentzian model with one break and with two breaks proposed by Hess and Soom are show in Eq. 3.9& 3.10 respectively<sup>[18]</sup>:

$$F(\dot{x}) = F_k + \frac{F_s - F_k}{1 + (\dot{x}/\dot{x}_s)^2} + F_v \dot{x}$$
(3.9)

$$F(\dot{x}) = F_k + \frac{F_s - F_k}{1 + (\dot{x}/\dot{x}_s)^2} + \frac{F_0 - F_k}{1 + (\dot{x}/\dot{x}_s)^2} + F_v \dot{x}$$
(3.10)

Regarding the presence of two break point, Armstrong believe<sup>[18]</sup>, this phenomenon is due to the topography difference between the two sliding point, although the sliding velocity is the same, different points may have different 'effective radii', thus different model need to be applied according to the state of motion.



Figure 3.9&3.10: Lorentzian model for Stribeck velocity set to be 0.1(left) Enlargement of the model in the boundary and partial fluid lubrication region (right)

Compare with Bo and Pavelescu's model, Lorentzian model has a relative strong constraint, only one parameter, Stribeck velocity can be adjusted. According to simulation results (Figure 3.10), Lorentzian model does show the 'relative constant velocity' in the boundary lubrication region.

#### 3.2.2 Model Selection

Bo and Pavelescu's exponential model (Eq. 3.7) is selected for the following reasons listed below:

1. The model is a nonlinear, it has a great flexibility in boundary and partial fluid lubrication region, all three curve types can be achieved by adjusting  $\sigma$  and.  $\dot{x}_s$ 

2. Consider the motion in experiments, circular motion is executed by a servomotor in Hess and Soom's experiment, while linear motion is suggested via Bo and Pavelescu's apparatus design (shown in A2).

3. Bo and Pavelescu's model is developed from data collected in steady state constant velocities motion, where the Lorentzian model based on the results of

closed-loop, changing velocity motion. In our experiment, data points are designed to be obtained from steady state constant velocity motion.

#### **3.2.3 Parameter justification**

Recall Eq. (7):  $F(\dot{x}) = F_k + (F_s - F_k)e^{(\dot{x}/\dot{x}_s)\sigma} + F_v \dot{x}$ , in our simulation scenario where:

$$F_{c} = \mu_{c}F_{n} \tag{3.11}$$

$$\mathbf{F}_{\mathrm{s}} = \boldsymbol{\mu}_{\mathrm{s}} \mathbf{F}_{\mathrm{n}} \tag{3.12}$$

$$F_v = 0.1$$
 (3.13)

The value of  $F_k$  and  $F_v$  are stick to the values which Azad and Featherstone <sup>[8]</sup> used in their modelling.  $\mu_s$  is set to be 0.24 due to the material properties of cork and steel. Regarding the value of  $\sigma$ , different people have different opinion. Bo and Pavelescu find  $\sigma$  is ranging between  $\frac{1}{2}$  and 1; Tustin<sup>[19]</sup> employed a value of 1 in his analysis of feedback control; the Gaussian model, with  $\sigma = 2$  is believed by Armstrong; from the experiment observation, Fuller<sup>[24]</sup> suggests that, if the boundary lubrication is effective, the magnitude of  $\sigma$  wound be very large. We set  $\sigma = 2$  to begin our simulation, thus, if the two bodies are in contact; the description of the friction model is therefore:

$$F_{f} = \begin{cases} sgn(F_{stick})F_{k} + sgn(F_{stick})(F_{s} - F_{c})e^{-\left(\frac{\dot{x}}{\dot{x}_{s}}\right)^{2}} + F_{v}\dot{x} & \text{if } \dot{x} \neq 0\\ sgn(F_{stick})F_{k} + sgn(F_{stick})(F_{s} - F_{c})e^{-\left(\frac{\dot{x}}{\dot{x}_{s}}\right)^{2}} + F_{v}\dot{x} & \text{if } F_{stick} > \mu_{s}F_{n} \\ f_{stick} & \text{otherwise,} \end{cases}$$
(3.14)

#### **3.3 Matlab Simulation and Results**

#### **3.3.1 Friction Force Simulation and Results**

Normalize  $F_f$  in Eq. 3.14, friction coefficient is obtained, Figure 3.11&3.12 shows friction coefficient vs. clutch velocity for model without and with Stribeck effect respectively. From these two diagrams, no difference can be told at this scale. Notice that both diagrams seem to be mirror symmetric versions of conventional f-v diagram (Figure 1.3b)), this is because the velocity of the sphere is set to be positive, clutch is designed to absorb energy and the motion of clutch would go against the motion of the whole sphere.



Figure 3.11&3.12: Friction coefficient vs. clutch velocity (model without Stribeck effect is on the left, model with Stribeck effect is on the right)

Zoom in Figure 3.11&3.12, Stribeck effect can be observed (Figure 3.13&3.14), not surprisingly, the additional Stribeck model does not have a great effect on the whole diagram.



Figure 3.13&3.14: Enlargement of Friction coefficient vs. clutch velocity (model without Stribeck effect is on the left, model with Stribeck effect is on the right)

Compare Figure 3.15&3.16 with Figure 2.7&2.8, we conclude that no significant increase in friction can be viewed by adding the Stribeck effect



Figure 3.15&3.16: Friction force in x direction (left) vs. time and Friction vs. time for the first bounce (right)

#### 3.3.2 Energy Audit

Figure 3.17 shows a total energy difference vs. time plot with Stribeck velocity. Via trial and error method, adjust the value of clutch velocity, we find that when it is equal to 0.03138 the overall energy difference is close to 0. From the diagram, positive value after the first impact indicates the whole system experiences some major energy loss in the first impact. The peak value of the energy loss occur in the first impact is  $1.5 \times 10^{-5}$  N·m, when the ball leaves the ground, the difference reduced to about  $0.5 \times 10^{-5}$  N·m. Afterwards, for each impact, the whole system gains energy gradually, ultimately, the total energy difference gets close to 0. However, the modelling of ground kinetic energy change is not incorporated into the simulation, which is reason for the whole system 'gaining energy' after each bounce. Compare Figure 3.18 with Figure 2.12&2.13 we can conclude that, contact model with Stribeck effect, gives a better simulation in energy audit.







Figure 3.18: Total energy different vs. time (with Stribeck effect, same scale with Fig. 2.12 and Fig 2.13)

## **Chapter 4 Experiment Design**

#### 4.1 Experiment aim

In Chapter 3, we have discussed a friction model which is not too complicated and able to capture most of the common friction effect and phenomena like Stribeck effect, pre-sliding displacement, viscous plus coulomb friction. In this section, we are going to introduce a scientific experiment method to obtain a friction vs. velocity diagram in a specific scenario: a steel cylinder sliding on a cork plate. Ultimately the accuracy and reliability of our mathematical model are mean to be testified by comparing the experiment data plot with simulated plot.

#### **4.2 Experiment Concept**

The major advantage of the 'improved' model is that: it incorporates the Stribeck effect in to the existing model. In order to observe Stribeck effect in the experiment, tools and equipment with high resolution are required. In designing a controlled experiment, minimizing the effects of variables other than the single independent variable is essential. This can be obtained by either good experiment setup or compensation method. If all elements in the experiment are well controlled, and major friction phenomena are captured, it is then possible to compare the experiment results with the simulated results.

In a specific frame of reference, motion, according to definition, can be described in terms of velocity, acceleration, displacement and time. Due to the different patterns in velocity, acceleration, motions can be classified as rotary, rolling, bouncing etc. Of all these motions, linear motion with zero acceleration is relatively easy in data collection and experiment controlling. In addition, to minimize the effect of friction lag, steady state constant velocity motion is selected for the experiment. This type of motion can be achieved by certain driving mechanisms, say, rotary motor and linear motor, as long as the machine is able to provide stable output. One thing need to be
point out is that, the fewer uncertainty and errors induced into the experiment, the higher precision and accuracy the experiment results will be.

There are two sets of data need to be measured in order for a friction vs. velocity diagram: one is velocity and the other is its corresponding friction force. In the consideration of high precision and accuracy, force data can be collected via force sensor, while the velocity information can be measure by linear encoder.

A9 shows the arrangement of the experiment. There are 5 major components: flat surface, driving mechanism, pulling object, linear encoder and force sensor. A simple description of experiment is: a fixed driving mechanism pulls a regular shape object via string on a flat surface with constant velocity, sensor and encoder measures experiment data and transmit the data to PC for computation.

#### 4.3 Experiment Tools and Materials Justification

#### **4.3.1 Pulling Object Selection**

Referring the material selected in the simulation section, the material selected for pulling object is steel, however, it is not necessary to apply steel for the whole object, a solid object with steel surface is sufficiently good for experiment purpose.

Regarding the shape of the object, the potential option could be sphere, cubic and cylinder. For Cubic, it is hard to fix one head of the pulling string along the neutral axis of the object and this may result in turbulent motion. For sphere, it is hard to stabilize its motion and rolling friction maybe observed which induces systematic errors into the system. Cylinder is the optimized solution, the shape guarantees the pulling string maintain along the natural axis when the motion is stabilized.

The larger the gap between  $F_k$  and  $F_S$  the easier the Stribeck effect can be measured. According to Eq.3.11 & 3.12, the value of friction coefficient difference and normal force determines the magnitude of the gap between  $F_k$  and  $F_S$ . The friction coefficients are constant which relate to mainly material properties, so change the size of mass would work here. Balance between experiment costs and performance, a steel cylinder with a mass of 10kg is selected. If the cylinder is purely steel, with a density of  $7.86 \times 10^3$  kg·m<sup>-3</sup>), its height and the radius are designed to be 0.0740m in the consideration of the force sensor mounting.

#### 4.3.2 Cork Plate

Experiment need to be carried out on a flat plate. In order be coherent with the simulation, the material selected for experiment is cork as well. The dynamic and static friction coefficients for steel sliding on cork plate are 0.20 and 0.24 respectively, thus the estimated difference between  $F_k$  and  $F_S$  3.92N and the expected maximum force ( $F_S$ ) is 23.54N.

Size of the cork plate is constraint by apparatus' dimension. After several experiment tools' dimensions are viewed <sup>[35], [42], [43]</sup>, cork plate is design to  $4 \times 1 \times 0.1$ m. (The 0.1m thickness is in the consideration of linear encoder installation)

#### 4.3.5 Back-of-the-Envelope Calculation and Justification

#### Force sensor and Linear Encoder Resolution Consideration

As mentioned in chapter 3, the slope of friction force vs. velocity curve would charge drastically within the Stribeck velocity region, thus the more velocity value taken within the Stribeck velocity region the better the Stribeck effect is described, and the more measurement taken for each velocity value point, the smaller the sample standard deviation is, ultimately, the more accurate the best fitted curve is. Take Figure 4.1 as an example, it shows an unsatisfactory model and experiment in two aspects: the model fit data bad and the lowest velocity data point is 2 standard deviation away from the best fitted curve in addition, when the velocities is extremely low, no measurements are made, however this velocity regime is important to force control. Figure 4.2 clearly shows a region of negative viscous friction by taking more values in the low velocity region and more measurements. In addition the best fitted curve go through most of the '90% confidence interval' as shown in the figure. One thing need to be point out is that, the experimental which produces Figure #1 and Figure #2 is carried out by a particular machine: puma 560 with spur gears and ball

bearing. The arm length of this type robot is 0.92 m (A 3), thus the place where  $0.05 \text{ rads}^{-1}$  is marked is very close to  $0.05 \text{ms}^{-1}$ 



Figure 4.1: Function of Velocity at Low Velocities, cure given by Tustin's Exponential

Model<sup>[18]</sup>



Figure 4.2: Friction as a Function of Velocity during Motions against a Complainant Surface <sup>[18]</sup>

In order to clearly show the negative slope at low velocity region and balance between accuracy of the model and expense, 10 velocity values are designed to be taken within the Stribeck velocity region. Magnitude of velocity need to be precisely controlled and the resolution of linear encoder should be able to identify 0.001ms-1 changes in velocity. Regarding the resolution of the force transducer, recall the Stribeck Model  $F(V)=F_k+(F_s - F_c)*e^{-(\frac{V}{Vs})^2} + F_v * V$ , Where:  $F_s = m^*g^*\mu_c = 10*9.81*0.24 = 23.54N$   $F_k = m^*g^*\mu_s = \!\!10^*9.81^*0.20 \!\!=\!\!19.62N$ 

Vs=0.05m/s

Fv=0.10

According to the friction difference between adjacent points calculated from the Stribeck model, the smallest value read from table 4.1 is -0.1142N, which indicates: the resolution of force transducer need to be close to 0.01N in order to detect a significant force variation when the speed is extremely low.

No.	V (ms-1)	<b>F</b> ( <b>V</b> ) ( <b>N</b> )	Friction Difference (N)
1	0.005	23.5015	NA
2	0.010	23.3873	-0.1142
3	0.015	23.2041	-0.1832
4	0.020	22.9624	-0.2417
5	0.025	22.6754	-0.2870
6	0.030	22.3579	-0.3175
7	0.035	22.0250	-0.3329
8	0.040	21.6910	-0.334
9	0.045	21.3683	-0.3227
10	0.050	21.0671	-0.3012

Table 4.1: Estimated friction for controlled velocities

#### Maximum velocity and length consideration

Consider the scale Figure 4.2, the velocity ranges from 0 ms<sup>-1</sup> to 0.12ms<sup>-1</sup> roughly. Multiply the maximum velocity by 10sec. 1.2m is obtained. Thus the length of the linear encoder, pulling string and track of linear motor should be no less than 1.2m. We can also adjust the velocity range due to the equipment limitations.

#### **4.3.6 Pulling String**

String with high tensile modulus is need is the experiment. Mechanical properties of yarns are shown in table 4.2.

Specifications	Specific	Modulus	Break	Specific Tensile Strength
	Density	(10 <sup>6</sup> psi)	Elongation	(10 <sup>6</sup> in.)
	(lb/in.3)		(%)	
Kevlar 29	0.052	10.2	3.6	8.15
Kevlar 49	0.052	18.0	2.4	8.37
S-Glass	0.090	12.4	5.4	7.40
E-Glass	0.092	10.5	4.8	5.43
Steel Wire	0.280	29	2.0	1.0
Nylon-66	0.042	0.8	18.3	3.40

 Table 4.2: Comparative properties of Kevlar vs. other yarns
 [31]

Kevlar 49 has a modulus of  $18.0 \times 10^6$  psi ( $1.24*10^5$  Mpa), comparing to other yarns like S-glass, E-glass and nylon66 Kevlar 49 has a significant advantage. Good resistance to elongation will minimize the errors induced in the system. One thing need to be pointed out is that the steel wire has an even higher tensile modulus  $(29 \times 10^6 \text{psi})$ , the reason of not choosing steel wire is because it may provide anti-compression force and turbulent the experiment data. So, Kevlar 49 is selected for the experiment.

#### 4.3.7 Force Transducer Selection

#### **Force Transducer Technology Selection**

The major functionality of the force transducer is to measure the force applied of the object. Force transducer is mounted on the pulling object and is connected to linear motor via Kevlar string (as shown in the A9), consider the free body diagram of cylinder, the magnitude of pulling the force should be equal to the friction force between the pulling object and cork plate. In order to describe the friction force thoroughly at low velocity region, a resolution of 0.01N is required. The experiment is design to be carried out in a stable environment, with normal atmospheric temperature  $(15-20^{\circ}C)$  and little temperature variation, thus charge mode system <sup>[32]</sup> and low

frequency response<sup>[33]</sup>, are not necessary consideration in the experiment.

Before the actual type of the force transducer is decided, the first step in the selection process is to determine the type of measurement to be made. Nowadays there four major force sensor technologies: piezoelectric, piezoresistive, inductive and capacitive. Compare all the four sensing principles, generally, piezoelectric sensor have a great advantage over others in sensitivity and resolution (table 4.3).

Principle	Strain Sensitivity (V/µ)	Threshold (μ*)	Span to threshold ratio
Piezoelectric	5.0	0.00001	100.000.000
Piezoresistive	0.0001	0.0001	2.500.000
Inductive	0.001	0.0005	2.000.000
Capacitive	0.005	0.0001	750.000

Table 4.3: Comparison of Sensing Principles <sup>[34]</sup>

In addition, piezoelectric force transducer has high modulus of elasticity; extremely high natural frequency and excellent linearity over wide amplitude range. These characteristics guarantees the accuracy of the data measured over a wide range of force implied.

The major disadvantage of piezoelectric sensors is that they do not have real static measurements, static behaviour can only be approximated. 'A static force will result in constant discharges on the piezoelectric material and yielding an inaccurate signal', according to 'piezocryst' <sup>[34]</sup>.

#### Solution to the constant discharge

Before we discuss the actual solution, the concepts of Discharge Time Constant need to be explained. 'It is the time (in seconds) required for a sensor output voltage signal to discharge 63% of its initial value immediately following the application of a long term, steady state input change', according to Basu<sup>[35]</sup>.

Apply the time constant analysis, the sensor piezoelctric element and internal integrated circuit amplifier may be represented schematically by the RC circuit, battery and switch shown in Figure 4.3. Gate voltage (V) responds is shown in Figure

4.4 and it can be interpreted by the Eq. 4.1.

$$V = V_0 * e^{(t/RC)}$$

$$\tag{4.1}$$

Where:

V=instantaneous gate voltage

 $V_0$ =initial voltage at time  $t_0$ 

e= base of natural logarithm

RC= time constant of RC discharging circuit



SWITCH CLOSES AT  $t = t_0$ 

Figure 4.3: Simplified RC circuit of Piezoelectric Force Sensor<sup>[36]</sup>



Figure 4.4: Discharge Time Constant (TC) Output vs. Time<sup>[37]</sup>

From Figure 4.4, the curve is relatively linear to about 10% TC. To say that exactly: in 1% of the discharge time constant, the sensor will discharge 1% and so on up to 10% of discharge time constant. Thus if we take the reading of the output within a time region of 1% of the sensor discharge time constant, the measurement inaccuracy due to constant discharge is negligible. Figure 4.5 shows the corresponding gate voltage. At instant time point  $t_0$ , voltage is assumed to have value  $V_0$ . After 1% of TC, voltage will decay in accordance with Equation 1, losing 1% of its initial value.



Figure 4.5: gate voltage discharge pattern up to 1% TC

Because the relationship between force measured and the voltage detected is linear, losing 1% of the initial voltage mean losing 1% of the force measured. In order to compensate the loss, first we need to measure the discharge time constant of a force sensor by counting the instant time that voltage/force drop to 37% of the initial value. We can repeat the measurement 10 times to reduce the negative effect of random variable. Find the average value of the discharge time constant (the value is generally very large around 1000s<sup>[36]</sup>) and multiply by 0.01, the 1% TC is obtained. Record the measured force at 1% TC and divide that number by 0.99, a more accurate friction force is obtained. The reason that taking force reading at 1% TC is because, 1% TC is reasonable time duration for motion to reach steady state.

#### **Model Selection**

The force sensor selected is a product of Dytran Instruments, Inc. The model number is 1050V1 (shown in A4) and the reason for the model chosen is listed below: 1. Suitable tension force measure range: the model has a tension range of 0-10Lbs (0-44.5N). The maximum estimated friction force occur in the experiment is not close to the force boundary, which guarantees the linear behaviour the force sensor.

2. Excellent weight and size: the sensor weights 28gs, with a size  $1.025in \times 0.625in \times 0.75in$  (2.60cm×1.59cm×1.9cm), which is suitable to be mounted on the pulling object. In addition, Force sensor is mounted to the cylinder via a

threaded stud.

3. Outstanding resolution and sensitivity: Sensor's sensitivity is  $500 \text{mVlbf}^{-1}$  and the resolution of the sensor is 0.00014 lb ( $6.47 \times 10^{-6}\text{N}$ ), which is excellent, however the force sensor is not functioning independently and a data-acquisition device is need to be companied with the force sensor in the experiment. As a matter of fact, to achieve the required resolution, a data-acquisition system with appropriate absolute accuracy is needed.

#### 4.3.8 Data-acquisition systems

There is no specific requirement for the Data-acquisition systems, as long as it provides a reasonable absolute accuracy and sample rate. The data acquisition model selected is NI USB-6008 provided by National Instruments <sup>[38]</sup> with a sample rate of 150Hz. The NI USB-6008 is low-cost data acquisition devices with easy screw connectivity, the USB port enables quick and easy installation to notebook and normal PC's., this devices is simple enough for quick measurements but versatile enough for more complex measurement applications.

Due to the result in the close envelop calculation, the expected peak force in measurement is 23.52N, thus the maximum expected voltage across the force sensor is 2.64V. For the data acquisition model, its absolute accuracy varies due to different input range, according to the data sheet (shown in A5), 2.64V lies inside the  $\pm$ 4V input range, the corresponding absolute accuracy is hence 3.59mV, and recall the sensitivity of the force sensor, which is 500mVlbf<sup>-1</sup>, the resolution of the whole system is calculated: 3.58/500lbf, convert to IS units is 0.032N.

#### 4.3.9 Linear Encoder

#### Linear Encoder Technology Section

An encoder is an electrical mechanical device that can monitor motion or position. There are a number of linear position technologies available in market, such as optical, magnetic, inductive, capacitive and eddy current, performance of linear encoders vary by stoke, resolution , cost, environmental resistance, as well as reliability. Understanding the differences between the sensing technologies can help in decision. Table 4.4 give a comparison of some major sensing technologies.

Characteristic	Potentiometer	AC-operated	Magnetostrictive	LVRTs	Optical	Magnetic
		LVDTs			Encoders	Encoders
Range	2.5-500mm	0.5-500mm	0.15-3.0m	0.25-10m	0.15-1.5m	0.15-3.0m
				m		
Accuracy	Moderate	Very Good	Excellent	Very Good	Excellent	Very Good
Resolution	Moderate	Excellent	Excellent	Excellent	Excellent	Very Good
Repeatability	Fair	Excellent	Excellent	Excellent	Very	Very Good
					Good	
Temperature	Fair	Excellent	Moderate	Excellent	Excellent	Good
Resistance						
Cost	Low	Moderate	High	Moderate	Moderate	Moderate
Linearity	Moderate	Good	Very Good	Good	Very	Very Good
					Good	

Table 4.4: Attribute comparison chart of the main linear position technology<sup>[39]</sup>

Consider our experiment requirement; encoder is designed to measure the instantaneous linear velocity of the moving cylinder with a resolution of at least 0.001m. Optical and magnetic encoders are suitable for our experiment setup, due to their excellent performance in linear motion measurement. For both encoders, micro level resolution is capable up to several meters per second <sup>[40]</sup> and the operation range of both encoders are more than one meter which is reasonable for this experiment purpose. However, regarding the arrangement of the encoder and the linear motor, the read head of the encoder is bind to the moving coil of the motor in a parallel manner, them are close to each other, that magnetic field generated by motor may some influence over the detected the magnetic signature of the magnetised scale. As a result,

optical linear coder is selected.

#### **Optical Encoder Technology**

Optical linear encoder measuring method could be either absolute or incremental, and modern optical technology includes self-imaging (imaging scanning) and interferometric <sup>[41]</sup>.

Figure 4.6 illustrates image Scanning Principle. Figure 4.7 presents interferential scanning principle. As can be seen, they have similar working principles: light-emitting diode (LED) is mounted on the read head of the encoder. The light emitted is continuously focused through a condenser lens and try to get through two scale gratings move relative to each other. If the lights successfully reach the phototransistor, which is mounted on the other side, signals are generated.

However, for interferential scanning principle, due to its diffraction and interference feature, 'their scanning signals are largely free of harmonics and can be highly interpolated. These encoders are therefore especially suitable for high resolution and high accuracy'.<sup>[42]</sup>



Figure 4.6& Figure 4.7: Image Scanning Principle <sup>[42]</sup> (left): Interferential scanning principle <sup>[42]</sup> (right)

#### **Model Selection**

Compared the price of optical linear encoder with different technology on website, such US DIGITAL and EBAY, for models with a measuring length of 1.2m, the prices are roughly around 2000 US dollars and there no distinct gap in price between products with different optical technology. The price is not a major factor in the model selection, thus product with best resolution, accuracy and smallest measurement error is selected. Table 4.5 shows some essential properties of optical linear encoder manufactured by Heidenhain; model LF183 with interferential scanning technology has an obvious superiority over other image scanning type models in nearly every aspects. The measurement length which I selected is 1640mm and the detail drawing of the model is shown in A6.

Figure 4.8 and 4.9 shows the position error over a measurement length (ML) and position error within a signal period respectively, these two curves can be applied to LF, LS, LB series model.  $\pm$ F are the extreme values which lie within the accuracy grade ( $\pm$ a). Inside the one-meter measuring length, the position error increase almost linearly, start from a negative value above –F and go through zero position error half way of total measuring length. According to Heidenhain <sup>[42]</sup>, 'The position error within one signal period is determined by the signal period of the encoder, as well as the quality of the graduation and the scanning process. At any measuring position, the position error does not exceed  $\pm$ 2% of the signal period. In addition, the smaller the signal period, the smaller the position error within one signal period.

Series of	Max. position error	Accuracy	Recommended	Required	Signal period of
model	u with one signal	grade	Resolution	moving force	scanning signals
	period				
LF 183	Approx. 0.08 µm	$\pm 2 \ \mu m, \pm$	0.1µm	$\leq 4N$	$\pm 2 \ \mu m$
		3 µm			
LS 487	Approx. 0.4 µm	$\pm$ 3 $\mu m, \pm$	0.5µm	$\leq 5N$	$\pm 20 \ \mu m$
		5 µm			

Table 4.5: Properties for series of mode <sup>[42]</sup>



Figure 4.8&4.9: Position error over the measuring length ML (left), Position error u within one signal period

#### 4.3.10 Driving mechanism

#### **Technology Selection**

The ideal motion for the experiment should provide the following features:

- 1. Able to pull the object continuously, with an extremely low velocity
- 2. Able to control the linear position, speed and acceleration precisely under loading
- 3. Vibration-free, Smooth linear motion over a reasonable range of velocities
- 4. Easy installation and works well with other experiment components
- 5. Low cost

In general, there are two types of driving mechanism which is suitable for our experiment, rotary motor and linear motor. Through both of them provides smooth, wide range of speed, and are able to able to maintain a low speed under loading condition, rotary motor needs an addition 'winch' mechanism to deal with the pulled string which will definitely induce some systematic error into the system (additional friction force maybe detected in the coiling process). For linear motor, the moving coil can be bonded to the mounting block, thus the motor can drive the cylinder and

the mounting block at the same velocity. For rotary motor, it is relatively harder to make the same achievement. We can bond the cylinder and mounting block together, however, two more errors are induced by this experiment set up: one is due to contact between cylinder and the housing of the encoder, the other raise from the friction in moving the mounting block. Due to the reason above, linear motor is selected.

#### **Model Selection**

Products of Yaskawa are primarily surveyed. There are three linear motor series and some important specifications of models from different series are list in table 4.6. Generally, all models selected have similar force constant suggest the linear motor have similar sensitivity to current changes. Models with large continuous force and large moving coil mass indicate that SGLTW series is designed for driving heave loading. With a suitable continuous force and low moving coil mass, SGLGW series seems to be a good choice, but referring to the design of SGLGW (A7), the magnet track is 62mm already, which is a way too high. Once the coil is incorporated into the system the total height goes to 77mm, if a string connects the coil and cylinder in a manner parallel to the ground, turning moment will arise (consider the shape of the cylinder). 120AII from SGLFW series perfectly meets the experiment requirement in force, size and shape (A8), besides, according to its design specification: 'Edge cogging is cancelled by magnetic force of sub-teeth <sup>[43],</sup>, which guarantees smooth, vibration linear motion.

Specification		SGLGW-40A		SGLF	SGLFW-35A		SGLTW-20A	
		140B	253B	120	230	170	320	
		II	II	AII	II	AII	AII	
Continu	Ν	47	93	80	160	130	250	
ous Force								
Continu	A <sub>RMS</sub>	0.8	16	1.4	2.8	2.3	4.4	

Table 4.6: Specifications of Yaskawa's products <sup>143</sup>	Table 4.6: S	pecifications	of Yaskawa's	products	[43]
--	--------------	---------------	--------------	----------	------

ous current	-						
Peak	Ν	140	280	220	440	380	760
Force							
Peak	A <sub>RMS</sub>	2.4	4.9	4.4	8.8	7.7	15.4
Current							
Moving	kg	0.39	0.65	1.3	2.3	2.6	4.6
Coil Mass							
Force	N/	61.5	61.5	62.4	62.4	61.0	61.0
Constant	A <sub>RMS</sub>						

#### **Motion Controller**

SGLFW series motors are design to be used with SGDH Sigma II Motion Controller. In order to make sure the whole system is able to maintain the velocity at an extremely low level, there is a specific term which needed to be look at: speed control range. It is the specification which explains the slowest velocity which the system can run and maintain control without overheating the motor with a full load. According to the given data sheet <sup>[44]</sup>, speed control range of SGDH Sigma II is 1: 5000. Call the Speed and force character diagram of SGLFW-35A 120A (Figure 4.10). Assume the motor always run at its maximum speed 5ms<sup>-1</sup>, divided by 5000 the lowest controlled velocity is 0.001 is obtained. Consider the experiment design, the smallest velocity increment is 0.005, thus the driving mechanism system meet the requirements



Figure 4.10: Speed and force character diagram<sup>[43]</sup>

### **4.4 Experiment Procedure**

- 1. Equipment Tools Preparation:
- (1) Kevlar 49 string with a length of 1.7m
- (2) Steel, cylinder shape pulling object
- (3) Flat corking plate with a dimension of  $4 \times 1 \times 0.1$
- (4) Linear motor SGLFW-35A<sup>2</sup>-120A•
- (5) Force Transducer, model 1053V1
- (6) Linear encoder model LF 183
- (7) Data-acquisition system model NI USB-6008
- (8) Motion controller model SGDH Sigma II
- 2. Equipment Installation and Mounting
- (1) Linear Encoder
  - Check the parallel deviation of the mounting surface, make sure the parallel deviation is less than 0.1mm, otherwise a mounting bracket is required
  - > Mark out the mounting holes, make sure they are evenly spaced
  - > Drill holes in the mark place with diameter of 4.3mm
  - Mount the linear encoder on the edge of the cork plate (A9), the top of the mounting block is 3cm above the plate.

- Use a dial-gauge to check the parallelism of the mounted module and adjustment is required when necessary
- (2) Linear Motor
  - Check the flatness of the plate surface
  - Mark the place for motor to be fixed on table, according to the mounting hole which the model provided. Make sure the track of the linear motor is 50mm away from the housing of the linear encoder.
  - > Drill holes in the mark place with diameter of 4.3mm
  - ➢ Fix the linear motor on plate via screws.
  - Check the parallelism of both linear motor and linear encoder again, adjustment is required when necessary, and guarantee the parallel deviation is less than 0.1mm
  - Place 2 nuts on moving coil via adhesive method and the space between the nuts is 40.0mm.
  - Connect the moving coil of linear motor to encoder's mounting block via two sets of special screws and nuts shown in A9. The screw has a diameter of 2.0mm and a length of 91.5mm. Make sure there is no relative motion between mounting block and moving coil and mounting block.

(3) Force Transducer

- > Weld a small O-ring at the centre point of the impact cap
- Weld another O-ring at the front of the moving coil. The welding point is 19.8mm above the motor track, along the neutral axis of the moving coil.
- Drill a hole on the front surface of the cylinder, with dimension specified in A4, when the cylinder stands on the plate, the centre of the hole is 30.0mm above the ground
- > Plug the force transducer into the pre-made hole using a stud
- Tie both head of the string on the different O-rings, and fasten is using a knot.
   Make sure the sting is parallel to the ground when pulled tighen.

#### (4) Devices Connection

Set connection between force transducer and data-acquisition system

- > Connect linear motor and linear encoder to motion controller
- Connect motion controller and data-acquisition system to PC, thus velocity can be controlled and vecocity and the corresponding friction can be recorded as planed.

#### 3. Pre-lab Measurements and Estimation

(1) Discharge Time Constant

- Find known mass (say, 2kg exactlly) counterpoise is required.
- Fix the force transducer at the bottom surface of the cork plate via stud.
- Carefully and slowly hook the counterpoise on the O-ring
- > Mark the time instant when the measured force is exactly equal to  $2.00 \times 9.81$ .
- > Mark the time instant when the measured force is exactly equal to  $1.62 \times 9.81$
- The difference of the two time instant is the discharge time constant. Repeat the measurement 10 times to reduce the possible random errors in the measurement.
- (2) Estimate the Elongation of the Kevlar 49
  - The initial length of Kevlar 49 is 1.7m, the friction force at steady state motion is known, the enlongation can be estimated according to the Kevlar 49 module information provided in tabel 4.2
  - Estimate the total displacement of the pulling object in the initial 10 seconds.
  - Compare the esimate enlongation data with the the esitimated diaplacement
- 4. Operation and Data Collection

The experiment is based on the assumption that the motion reaches stable state at the beginning of the 10th second.

(1) The Sting should be tightened and the cylinder and the track should be along the same line

- (2) Check all the connection and powers
- (3) Set the velocity of linear motor to be 0.005 for first measurement
- (4) Turn on the linear motor; let it run for 11s, collect the data from the beginning of
- the 10<sup>th</sup> second to the end of the 10<sup>th</sup> second (which is solid 1second time duration).
- (5) Repeat the experiment for the same velocity 10 times and collect data

(6) Increase the velocity by 0.005 and repeat step (3), (4), (5) until the velocity reaches  $0.12 \text{ms}^{-1}$ .

(7) Use the measured discharge time constant, apply the method introduced in 4.3.6, and find the 'actual' friction force. (Since optical linear encoder has a much higher the resolution then the required level, the raw data obtained by encoder can be used for analysis directly.)

(8) Find the standard deviations of the friction force data for each velocity point

(9) Use the obtained standard deviations construct a 90% confidence interval bars.

(10) Plot the best fit curve and make sure that the curve go through most of the 90% confidence interval bars.

(11) Compare the plot with the force vs. velocity diagram which obtained from simulation.

### **Chapter 5 Conclusion**

#### 5.1 Outcomes

This aim of this thesis is to refine Azad and Featherstone's contact model in low relative velocity region. In their original friction model, most common friction effects and Phenomena like pre-sliding regime and coulomb + viscous friction are covered, however, with the aid of Matlab, it is found that Azad and Featherstone's model have a overall negative value in energy audit simulation.

Regarding this issue, after several models are viewed, an addition model which covers Stribeck effect is selected. The incorporation of this addition term gives a better energy audit simulation result by varying the empirical constant term  $\dot{x}_s$ . Though the overall value of the difference between the initial system energy and the total system energy after impact is very close to zero, the whole system do gain energy after each contact.

To testify the accuracy and reliability of the improved model, a scientific experiment design is introduced. Based on the results of a back-of-the-envelope calculation, tools and apparatus are justified and selected. Detailed experiment concepts and procedures are illustrated.

#### **5.2 Further Work**

Notice, for the simulation part, the whole system gains energy after each contact; this is due to the absence of the kinetic and potential energy consideration in cork plate. To further improve the model in the future, we can take these factors in to account.

An experiment design has been proposed, but due to time limitation, lubrication condition and lubricates discussion are excluded in the thesis. In addition, designed experiment is not carried out and the accuracy and reliability of the improved model is not testified.

# Appendix



## A1 Hess and Soom's Apparatus; (a) front view, (b) side view <sup>[22]</sup>



F

# A2. Bo and Pavelescu's Apparatus<sup>[27]</sup>



#### A3 Parmeters of the PUMA 560[]



#### A4: Data sheet of Force Transducer, model 1053V1



#### A4 continue

#### SPECIFICATIONS MODEL SERIES 1053V DYNAMIC FORCE SENSORS

#### SPECIFICATIONS BY MODEL

MODEL	SENSITIVITY +/-10%	COMPRESSION RANGE	MAXIMUM COMP.	TENSION RANGE		DISCH. TC	
12 13	(MV/LD)	(LDS)	(LDS)	(LDS)	(LDS) [1]	(Sec)	(LD, RIVIS)
1053V1	500	10	200	10	200	50	.00014
1053V2	100	50	1000	50	200	100	.0007
1053V3	50	100	2000	100	200	500	.0014
1053V4	10	500	10,000	200	200	2000	.007
1053V5	5	1000	15,000	200	200	2000	.014
1053V6	1	5000	15,000	200	200	2000	.07

#### COMMON SPECIFICATIONS

SPECIFICATION	VALUE	UNITS		
STIFFNESS	11.4	Lb/μ In		
MOUNTED RESONANT FREQUENCY, UNLOADED	75	kHz		
LINEARITY [2]	+/- 1	%F.S.		
F.S.OUTPUT VOLTAGE, NOM.	5	VOLTS		
MAX SHOCK, UNLOADED	10,000	G's		
MAX. VIBRATION, UNLOADED	+/- 5,000	G's		
COEFFICIENT OF THERMAL SENSITIVITY	.03	%/ <sup>0</sup> F		
TEMPERATURE RANGE	-100 to +250	٥F		
ENVIRONMENTAL SEAL	EPOXY			
SUPPLY CURRENT / VOLTAGE RANGE [3] 2 to 20 / +1	8 to +30	mA / VDC		
OUTPUT IMPEDANCE	100	OHMS		
MATERIAL	STAINLESS STEEL			
WEIGHT	28	GRAMS		
MOUNTING PROVISION 10-32 x .175 DEEP TAPPED HOLE IN TOP AND BOTTOM SURFACES				
ELECTRICAL CONNECTOR, RADIAL	10-32	COAXIAL		

#### ACCESSORIES SUPPLIED: (1) MOD 6213 STEEL IMPACT CAP, (2) MOD 6562 10-32 MOUNTING STUD

#### [1] Absolute maximum tension. Do not exceed in any case!

- [2] Percent of full scale or of any lesser range, zero based best fit straight-line method.
- [3] Power these instruments **only** with constant current type power units. **Do not** connect to a source of voltage without current limiting. This **will destroy** the integral IC amplifier.

Absolute accuracy at full scale, differential <sup>4</sup>					
Range	Typical at 25 °C (mV)	Maximum over Temperature (mV)			
±20	14.7	138			
±10	7.73	84.8			
±5	4.28	58.4			
±4	3.59	53.1			
±2.5	2.56	45.1			
±2	2.21	42.5			
±1.25	1.70	38.9			
±1	1.53	37.5			

# A5 Absolute accuracy table for NI USB-6008 <sup>[40]</sup>

### A6: Detailed Drawing for linear encoder model LF 183<sup>[42]</sup>

## LF 183

- Incremental linear encoder for measuring steps to 0.1 µm
- High positioning accuracy through single-field scanning Thermal behavior similar to steel or cast iron .
- High vibration rating
- Horizontal mounting possible







Dimensions in mm

Tolerancing ISO 8015

ISO 2768 - m H < 6 mm: ±0.2 mm 0, 0,

Image: Image:

- Woohling options
   F = Machine guideway
   P = Gauging points for alignment
   © = Required mating dimensions
   © = Compressed air inlet
   @ = Reference mark position on LF 183
   © = Reference mark position on LF 183 C
- (5) = Beginning of measuring length (ML)

⇔ = Direction of scanning unit motion for output signals in accordance with interface description

# A7: CAD Drawing For Linear Motor SGLFW-35A 120AII <sup>[43]</sup>





### A8: CAD Drawing For Linear Motor SGLGW-40A 140B<sup>[43]</sup>

## A9 Sketch for Apparatus



#### A10. Matlab Codes:

```
function sphere3DEnergyAudit
% sphere3DEnergyAudit simulate sphere motion (demonstrating of bouncing
and
% rolling motion of a sphere on the ground using our new contact model).
It
% also is able to audit the energy during the whole motion of the sphere.
% constants and sphere parameters
g = 9.81;
m = 0.154;
r = 0.0165;
I = 2/5 * m * r^2;
regimestate='n';
% initial conditions
qinit = [ 0 0 0.1 0 0 0]; % falling from 0.1m height
qdinit = [ 0.5 0.5 0 0 0 0]; % nonzero initial velocities of the sphere
ux0 = 0; % initial tangential deformation of the ground in x direction
uy0 = 0; % initial tangential deformation of the ground in y direction
% euler angles (only for use in showmotion)
phi0 = 0;
theta0 = 0;
psi0 = 0;
initcond = [ qinit qdinit ux0 uy0 phi0 theta0 psi0];
options = odeset( 'RelTol', 1e-9, 'AbsTol', ones(1,17) * 1e-12, ...
   'MaxStep', 1e-3 );
[T, Z] = ode45( @odefunc, [0 1], initcond, options );
% force calculation to the aim of plotting
z = Z(:,3);
xdot = Z(:, 7);
ydot = Z(:, 8);
zdot = Z(:, 9);
omegax = Z(:, 10);
omegay = Z(:, 11);
omegaz = Z(:, 12);
u = Z(:, 13:14)';
\$uy = Z(:, 14);
def = z - r;
V=zeros(2, length(T));
```

```
for i = 1: length(T)
   fn(i) = fnormal(z(i)-r, zdot(i));
   if (fn(i) > 0)
       V(:,i) = [xdot(i) - r*omegay(i); ydot(i) + r*omegax(i)];
        Vx(i) = xdot(i) - r*omegay(i);
8
8
        Vy(i) = ydot(i) + r*omegax(i);
   else
        V(:,i) = [0;0];
8
      Vx(i) = 0;
      Vy(i) = 0;
   end
   if i==1
    [ ft, udot, regimestate(i)] = ftangent3D( def(i), zdot(i), u(:,i),
V(:,i), fn(i), regimestate );
   else
   [ ft, udot, regimestate(i)] = ftangent3D( def(i), zdot(i), u(:,i),
V(:,i), fn(i), regimestate(i-1) );
   end
00
     [ ftx(i), fty(i), uxdot(i), uydot(i)] = ftangent3D( def(i),
zdot(i),...
        ux(i), uy(i), Vx(i), Vy(i), fn(i) );
8
   ftx(i) = ft(1);
   fty(i) = ft(2);
   uxdot(i) = udot(1);
   uydot(i) = udot(2);
end
def = def .* (def < 0);
% energy audit
x = Z(:, 1);
y = Z(:, 2);
Kn = 8.5e6;
for i = 1:length(T)
   Ebody(i) = m^*g^*z(i) + 1/2^*m^*(xdot(i)^2+ydot(i)^2+zdot(i)^2) + ...
      1/2*I*(omegax(i)^2+omegay(i)^2+omegaz(i)^2);
   ENSp(i) = 2/5*Kn*sqrt(abs(def(i))^5);
```

```
[NDamp(i) tDamp clutch tSp] = EnergyAuditIntegrands( def(i),
zdot(i), ...
      u(:,i), V(:,i) );
   TDamp(:,i) = tDamp;
   Clutch(:,i) = clutch;
   TSp(:,i) = tSp;
   if i==1
      ENDamp(i) = NDamp(i);
      ETDamp(:,i) = TDamp(:,i);
      EClutch(:,i) = Clutch(:,i);
      ETSp(:,i) = TSp(:,i);
   else
      dt = T(i) - T(i-1);
      ENDamp(i) = ENDamp(i-1) + (NDamp(i-1)+NDamp(i))/2*dt;
      ETDamp(:,i) = ETDamp(:,i-1) + (TDamp(:,i-1)+TDamp(:,i))/2*dt;
      EClutch(:,i) = EClutch(:,i-1) +
(Clutch(:,i-1)+Clutch(:,i))/2*dt;
      ETSp(:,i) = ETSp(:,i-1) + (TSp(:,i-1)+TSp(:,i))/2*dt;
   end
end
PotE = Ebody + ENSp + ETSp(1, :) + ETSp(2, :);
DampedE = ENDamp + ETDamp(1, :) + ETDamp(2, :) + EClutch(1, :) + EClutch(2, :);
ETotal = PotE + DampedE;
응응응응응
Einitial=g*m*0.1+0.5*m*(norm([0.5 0.5]))^2;
testenergy=Einitial-ETotal;
% plotting the rsults
plotresults (T, Z, def, fn, ftx, fty, Ebody, ENSp, ENDamp, ETDamp, ...
   EClutch, ETSp, PotE, DampedE, testenergy );
% animation
Q = [Z(:, 1:3) Z(:, 15:17)];
model = rolsphere;
showmotion( model, T, Q' );
   function dqdt = odefunc(t, q)
      % calculating values of ordinary differential equations
      % states
      x = q(1);
      y = q(2);
```

```
z = q(3);
      theta1 = q(4);
      theta2 = q(5);
      theta3 = q(6);
      xdot = q(7);
      ydot = q(8);
      zdot = q(9);
      omegax = q(10);
      omegay = q(11);
      omegaz = q(12);
      ux = q(13);
      uy = q(14);
      phi = q(15);
      theta= q(16);
      psi = q(17);
      u = [ux; uy];
      fn = fnormal( z-r, zdot ); % normal contact force
      % velocity of the contact point on the sphere
      if (fn > 0)
          V = [xdot - r*omegay; ydot + r*omegax];
      else
         V = [0; 0];
      end
       % tangential forces
       [ ft, udot, regimestate] = ftangent3D( z-r, zdot, u, V, fn,
regimestate);
      ftx = ft(1); fty = ft(2);
      qdd = dynamics( ftx, fty, fn );
      % euler angles
      thetadot = omegax*sin(psi) + omegay*cos(psi);
       if (\cos(\text{theta}) == 0)
          phidot = 0;
          psidot = 0;
      else
          phidot = (omegax*cos(psi) - omegay*sin(psi))/cos(theta);
          psidot = (-omegax*cos(psi) +
omegay*sin(psi))*sin(theta)/cos(theta);
      end
```

```
dqdt = [ q(7:12); qdd; udot; phidot; thetadot; psidot];
end
function qdd = dynamics( ftx, fty, fn )
   % dynamics calculations
   % momentum about the CoM of the sphere
   momentumx = fty * r;
   momentumy = -ftx * r;
   % motion equations
   xdd = ftx / m;
   ydd = fty / m;
   zdd = fn / m - g;
   alphax = momentumx / I;
   alphay = momentumy / I;
   alphaz = 0;
   qdd = [ xdd; ydd; zdd; alphax; alphay; alphaz];
end
function plotresults (T, Z, def, fn, ftx, fty, Ebody, ENSp, ENDamp, ...
      ETDamp, EClutch, ETSp, PotE, DampedE, testenergy )
   % plotting the results
   % plotting the states
   figure;
   subplot(3, 2, 1);
   plot( T, Z(:,1), 'k');
   xlabel( ' Time (s) ' );
   ylabel( ' x (m) ' );
   subplot(3,2,2);
   plot( T, Z(:,7), 'k');
   xlabel( ' Time (s) ' );
   ylabel( ' xdot (m/s) ' );
   subplot(3,2,3);
   plot( T, Z(:,2), 'k');
   xlabel( ' Time (s) ' );
   ylabel( ' y (m) ' );
```

```
subplot(3,2,4);
plot( T, Z(:,8), 'k');
xlabel( ' Time (s) ' );
ylabel( ' ydot (m/s) ' );
subplot(3,2,5);
plot( T, Z(:,3), 'k');
xlabel( ' Time (s) ' );
ylabel( ' z (m) ' );
subplot(3,2,6);
plot( T, Z(:,9), 'k');
xlabel( ' Time (s) ' );
ylabel( ' zdot (m/s) ' );
figure;
subplot(3,2,1);
plot( T, Z(:,4) * 180 / pi, 'k');
xlabel( ' Time (s) ' );
ylabel( ' \theta1 (degree) ' );
subplot(3,2,2);
plot( T, Z(:,10), 'k');
xlabel( ' Time (s) ' );
ylabel( ' \thetaldot (rad/s) ' );
subplot(3,2,3);
plot( T, Z(:,5) * 180 / pi, 'k');
xlabel( ' Time (s) ' );
ylabel( ' \theta2 (degree) ' );
subplot(3,2,4);
plot( T, Z(:,11), 'k');
xlabel( ' Time (s) ' );
ylabel( ' \theta2dot (rad/s) ' );
subplot(3, 2, 5);
plot( T, Z(:,6) * 180 / pi, 'k');
xlabel( ' Time (s) ' );
ylabel( ' \theta3 (degree) ' );
subplot(3,2,6);
plot( T, Z(:,12), 'k');
```
```
xlabel( ' Time (s) ' );
ylabel( ' \theta3dot (rad/s) ' );
% plotting the ground deformations
figure;
subplot(3,1,1);
plot( T*1000, Z(:,13), 'k');
xlabel( ' Time (s) ' );
ylabel( ' Time (s) ' );
% hold on;
% plot([130 134], [0 0]);
% axis([130 134 -5 12]);
```

```
subplot(3,1,2);
plot( T*1000, Z(:,14), 'k');
xlabel( ' Time (s) ' );
ylabel( ' uy (m) ' );
```

```
subplot(3,1,3);
plot( T*1000, def*1000, 'k');
xlabel( ' Time (s) ' );
ylabel( ' z (mm) ' );
% hold on;
% plot([130 134], [0 0]);
% axis([130 134 -1 0.5]);
```

```
% plotting the contact forces
figure;
subplot(3,2,1);
plot( T, fn, 'k');
xlabel( ' Time (s) ' );
ylabel( ' Normal Force (N) ' );
```

```
subplot(3,2,2);
plot(T*1000, fn, 'k');
xlabel(' Time (ms) ' );
ylabel(' Normal Force (N) ' );
axis([130 134 0 250]);
```

```
subplot(3,2,3);
plot( T, ftx, 'k');
xlabel( ' Time (s) ' );
```

```
ylabel( ' Fx (N) ' );
subplot(3,2,4);
plot( T*1000, ftx, 'k');
xlabel( ' Time (ms) ' );
ylabel( ' Fx (N) ' );
axis([130 134 -40 15]);
subplot(3,2,5);
plot( T, fty, 'k');
xlabel( ' Time (s) ' );
ylabel( ' Fy (N) ' );
subplot(3,2,6);
plot( T*1000, fty, 'k');
xlabel( ' Time (ms) ' );
ylabel( ' Fy (N) ' );
axis([130 134 -40 15]);
figure;
subplot(3,2,1);
plot(T, Ebody, 'k');
xlabel(' Time(s) ');
ylabel(' Energy of the Body (N.m)');
subplot(3,2,2);
plot(T, ENSp, 'k');
xlabel(' Time(s) ');
ylabel(' Normal Springs Energy (N.m)');
subplot(3,2,3);
plot(T, ENDamp, 'k');
xlabel(' Time(s) ');
ylabel(' Normal Dampers Energy (N.m)');
subplot(3,2,4);
plot(T, ETDamp(1,:)+ETDamp(2,:), 'k');
 hold on
 plot(T, ETDamp(2,:), 'k');
xlabel(' Time(s) ');
ylabel(' Tangential Dampers Energy (N.m)');
```

00

8

```
subplot(3,2,5);
      plot(T, EClutch(1,:)+EClutch(2,:), 'k');
        hold on
00
90
        plot(T, EClutch(2,:), 'k');
      xlabel(' Time(s) ');
      ylabel(' Energy of the Clutches (N.m)');
      subplot(3,2,6);
      plot(T, ETSp(1,:)+ETSp(2,:), 'k');
8
        hold on
%
        plot(T, ETSp(2,:), 'k');
      xlabel(' Time(s) ');
      ylabel(' Tangential Springs Energy (N.m)');
       figure;
      subplot(3,1,1);
      plot(T, PotE, 'k');
      xlabel(' Time(s) ');
      ylabel(' Potential Energy (N.m)');
      subplot(3, 1, 2);
      plot(T, DampedE, 'k');
      xlabel(' Time(s) ');
      ylabel(' Damped Energy (N.m)');
      subplot(3,1,3);
      plot(T, testenergy, 'k');
      xlabel(' Times(s) ');
      ylabel('Energy Difference (N.m) ');
8
        axis([0 1 0.18 0.2]);
8
        figure;
00
        plot( T, Z(:,1), 'k');
%
        hold on;
8
        plot( T, xdot - r*omegay);
        xlabel( ' Time (s) ' );
9
90
        axis([0 1 -0.2 0.55]);
```

end

end

```
function [ ft, udot, regimestate] = ftangent3D( z, zdot, u, V, fn,
regimestate )
% ftangent3D calculate tangent contact forces in 3D
% ftangent3D calculate tangent contact forces (ftx and fty) and
% tangential velocities (uxdot and uydot) of the contact point on the ground
(not the
% body) using our new contact model. z is the ground deformation and zdot
% is the rate of its change both in the normal direction. ux and uy are
% ground deformations in the tangential directions,
% Vx and Vy are the velocities of the contact point (on the body) in the
tangential
% directions and fn is the normal contact force.
   Kt = 12.75e6; % stiffness coef.
   Dt = 3.1e3;
                 % damping coef.
                % friction coef.
   mu = 0.2;
   mu2=0.201;
   Cv=0.1;
   % | fstick | £½sqrt(fx*fx+fy*fy)
   fstick = -Kt * abs(z)^{(1/2)} * u - Dt * abs(z)^{(1/2)} * V;
   fslip = mu * fn * fstick / norm(fstick);
% fs=norm(fstick);
   fs=fn*0.205;
   fc=fn*0.2;
                 %Fc=Fn*mu
   e=2.71828;
   Vs=0.048;
   if (fn>0)
      switch (regimestate)
          case {'k','n'}
             if fstick>(mu2*fn)
f1=@(Vclutchx)(V(1)+((fslip(1)+sign(fstick(1))*((fs-fc)*e^(-(abs(Vclu
tchx/Vs))^2))-Cv*Vclutchx)+Kt*abs(z)^(1/2)*u(1)-Cv*Vclutchx)/(Cv+Dt*a
bs(z)^{(1/2)} - Vclutchx;
```

```
Vclutchx=fzero(f1,0.05);
```

```
f2=@ (Vclutchy) (V(2)+((fslip(2)+sign(fstick(2))*((fs-fc)*e^{(-(abs(Vclutchy/Vs))^2))-Cv*Vclutchy)+Kt*abs(z)^{(1/2)}u(2)-Cv*Vclutchy)/(Cv+Dt*abs(z)^{(1/2)}))-Vclutchy;
```

```
Vclutchy=fzero(f2,0.05);
```

```
fstribeckx=fslip(1)+sign(fstick(1))*((fs-fc)*e^(-(abs(Vclutchx/Vs))^2
))-Cv*Vclutchx;
fstribecky=fslip(2)+sign(fstick(2))*((fs-fc)*e^(-(abs(Vclutchy/Vs))^2
))-Cv*Vclutchy;
                 ft=[fstribeckx; fstribecky];
                 udot = (-(ft + Kt * abs(z)^{(1/2)} * u) + Cv*V) / (Cv +
Dt * abs(z)^{(1/2)};
                 Vclutch=[Vclutchx; Vclutchy];
                 regimestate='p';
             else
                 ft=[fstick(1); fstick(2)];
                 udot = -(ft + Kt * abs(z)^{(1/2)} * u) / (Dt * abs(z)^{(1/2)});
                 Vclutch=[0; 0];
                 regimestate='k';
             end
          case 'p'
f1=@(Vclutchx)(V(1)+((fslip(1)+sign(fstick(1))*((fs-fc)*e^(-(abs(Vclu
tchx/Vs))^2))-Cv*Vclutchx)+Kt*abs(z)^(1/2)*u(1)-Cv*Vclutchx)/(Cv+Dt*a
bs(z)^{(1/2)} - Vclutchx;
                Vclutchx=fzero(f1,0.05);
```

```
f2=@(Vclutchy)(V(2)+((fslip(2)+sign(fstick(2))*((fs-fc)*e^(-(abs(Vclu
tchy/Vs))^2))-Cv*Vclutchy)+Kt*abs(z)^(1/2)*u(2)-Cv*Vclutchy)/(Cv+Dt*a
bs(z)^(1/2)))-Vclutchy;
Vclutchy=fzero(f2,0.05);
```

```
fstribeckx=fslip(1)+sign(fstick(1))*((fs-fc)*e^(-(abs(Vclutchx/Vs))^2
```

```
))-Cv*Vclutchx;
```

```
fstribecky=fslip(2)+sign(fstick(2))*((fs-fc)*e^(-(abs(Vclutchy/Vs))^2))-Cv*Vclutchy;
```

```
ft=[fstribeckx; fstribecky];
```

```
udot = (-(ft + Kt * abs(z)^{(1/2)} * u) + Cv*V) / (Cv + U)
Dt * abs(z)^{(1/2)};
                 %Vclutch=[Vclutchx Vclutchy];
              if fstick>(mu*fn)%norm(Vclutch)>0
                 ft=[fstribeckx; fstribecky];
                 Vclutch=[Vclutchx; Vclutchy];
                 udot= (-(ft + Kt * abs(z)^{(1/2)} * u) + Cv*V) / (Cv + Dt)
* abs(z)^(1/2));
                 regimestate='p';
              else
                 ft=[fstick(1); fstick(2)];
                 udot = -(ft + Kt * abs(z)^{(1/2)} * u) / (Dt * abs(z)^{(1/2)});
                 Vclutch=[0; 0];
                 regimestate='k';
             end
       end
   else
       if (z < 0) \,\% no contact but the ground is still recovering
          ft = [0; 0];
          udot = u/z * zdot;
          Vclutch=[0; 0];
          test22=0;% recovery rate
          regimestate='n';
       else
          ft = [0; 0];
          udot = [0; 0];
          Vclutch=[0; 0];
          test22=0;
          regimestate='n';
       end
   end
     if (fn > 0) % in contact
8
8
         if (norm(fstick) > mu*fn) % slipping mode
            udot = (-(fslip + Kt * abs(z)^{(1/2)} * u) + Cv*V) / (Cv + Dt)
8
* abs(z)^(1/2));
            Vclutch = V - udot;
8
00
            ft = fslip - Cv*Vclutch;
```

```
8
      else % sticking mode
9
          ft = fstick;
8
           udot = -(ft + Kt * abs(z)^{(1/2)} * u) / (Dt * abs(z)^{(1/2)});
8
           Vclutch = [0; 0];
8
       end
8
    else
8
       Vclutch = [0; 0];
       if (z < 0) % no contact but the ground is still recovering
8
8
           ft = [0; 0];
           udot = u/z * zdot; % recovery rate
8
       else % no contact
8
          ft = [0; 0];
2
8
          udot = [0; 0];
8
       end
8
     end
  %test=norm(a) is used to find |Vclutch|
  %test=norm(a);
  ft;
  udot;
```

```
end
```

```
Fnormal.m
```

```
function fn = fnormal( z, zdot )
% fnormal calculate normal contact force
% fnormal calculate normal contact force (fn) using our new contact model.
% z is the ground deformation and zdot is the rate of its change both in
% the normal direction.
Kn = 8.5e6; % stiffness coef.
Dn = 3.1e3; % damping coef.
if (z <= 0) % in contact</pre>
```

```
fK = 0;
      fD = 0;
   end
   fn = fK + fD;
end
EnergyAuditIntegrands.m
%%But why the nSp is missing, is it suppose to the the nSp missing?negtive,
%%the nSp is calculated from other approach
function [nDamp tDamp clutch tSp] = EnergyAuditIntegrands( z, zdot, u,
V )
% EnergyAuditIntegrands calculate integrands to the aim of energy audit
% computations. nDamp, tDamp, clutch and tSp are integrands related to
% normal damper, tangential damper, clutch and tangential springs,
% respectively.
   Kn = 8.5e6; % stiffness coef.
   Dn = 3.1e3;
                 % damping coef.
   Kt = 12.75e6; % stiffness coef.
   Dt = 3.1e3; % damping coef.
   mu = 0.2;
                % friction coef.
   Cv = 0.1; % viscous friction coef.
   if (z <= 0) % in contact
      fK = Kn * abs(z)^{(3/2)};
                                                 % spring force
      fD = max( - Dn * abs(z)^{(1/2)} * zdot, -fK); % damper force
   else
                % no contact
      fK = 0;
      fD = 0;
   end
   fn = fK + fD;
   % calculating sticking forces in both tangential directions
   fstick = - Kt * abs(z)^{(1/2)} * u - Dt * abs(z)^{(1/2)} * V;
   % calculating slipping forces in both tangential directions
```

```
fslip = mu * fn * fstick / norm(fstick);
```

```
if (fn > 0) % in contact
      if (norm(fstick) > mu*fn) % slipping mode
         udot = (-(fslip + Kt * abs(z)^{(1/2)} * u) + Cv*V) / (Cv + Dt * U)
abs(z)^{(1/2)};
         Vclutch = V - udot;
         ft = fslip - Cv*Vclutch;
      else
                      % sticking mode
         ft = fstick;
         udot = -(ft + Kt * abs(z)^{(1/2)} * u) / (Dt * abs(z)^{(1/2)});
         Vclutch = [0;0];
      end
   else
      Vclutch = [0;0];
      if (z < 0) % no contact but the ground is still recovering
         ft = [0;0];
         udot = u/z * zdot; % recovery rate
      else % no contact
         ft = [0;0];
         udot = [0;0];
      end
   end
   nDamp = abs(fD*zdot);
   tDamp = Dt*sqrt(abs(z))*udot.^2;
   clutch = abs(ft.*Vclutch);
   tSp = Kt*sqrt(abs(z))*u.*udot; %%%?????correct?yes?
```

end

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